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THE COLLUSIVE EFFICACY OF COMPETITION CLAUSES IN BERTRAND MARKETS WITH CAPACITY-CONSTRAINED RETAILERS

Michael Trost
University of Hohenheim

Institute of Economics

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The Collusive Efficacy of Competition Clauses in Bertrand Markets with Capacity-Constrained Retailers*

Michael Markus Trost†

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Abstract

We study the collusive efficacy of competition clauses (CC) such as the meeting competition clause (MCC) and the beating competition clauses (BCC) in a general framework. In contrast to previous theoretical studies, we allow for repeated interaction among the retailers and heterogeneity in their sales capacities. Besides that, the selection of the form of the CC is endogeneized. The retailers choose among a wide range of CC types - including the conventional ones such as the MCC and the BCCs with lump sum refunds. Several common statements about the collusive (in)efficacy of CCs cannot be upheld in our framework. We show that in the absence of hassle costs, MCCs might induce collusion in homogeneous markets even if they are adopted only by few retailers. If hassle and implementation costs are mild, collusion can be enforced by BCCs with lump sum refunds. Remarkably, these findings hold for any reasonable rationing rule. However, a complete specification of all collusive CCs is in general impossible without any further reference to the underlying rationing rule.

JEL–Classifications: L11, L13, L41

Keywords: Competition clauses, price-matching guarantee, price-beating guarantee, anti-competitive practice, capacity-constrained oligopoly

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†Institute of Economics, University of Hohenheim, trost.michael@uni-hohenheim.de.

1 Introduction

Low price guarantees in which the retailers promise to match or beat any lower price announced by a competitor are widely used in retail markets. Such guarantees have been popular in various market segments such as consumer electronics, automotive parts, DIY products, and glasses, to name only a few. Presumably, most readers of this paper could quote a retailer offering a low price guarantee.

Competition economists have already dealt with low price guarantees since the early 80s, starting with the seminal papers of Hay (1982) and Salop (1986). Since then, various explanations for these marketing practices have been provided and the question of whether such practices are a matter of competitive concern have run through many of those studies. Our paper aims to contribute to this still ongoing debate. In doing so, we adopt the terminology often applied in research and refer to low price guarantees as *competition clauses* (CC) from now on.

Most of the papers on this subject (in particular, the earlier papers like the ones of Hay, 1982, and Salop, 1986) are very skeptical regarding CCs. Despite of their pro-competitive appeal, such clauses are viewed as a cunning device facilitating tacit collusion. A situation in which the retailers charge an excessively high price for the commodity might be unsustainable in competitive markets as each retailer is tempted to slightly undercut the price in order to attract customers of its competitors. However, this incentive can be suspended if the competitors have adopted CCs; for example in form of price matching, also known as the meeting competition clause (MCC). A retailer could not profit by underselling its competitors in this case as their MCCs ensure that a lower price is immediately matched by them.

Although these arguments seem quite convincing, the claim that CCs are anti-competitive practices has come under severe attack for several reasons. One reason is that most of the studies backing the claim, e.g., Doyle (1988), Logan and Lutter (1989) and Zhang (1995), imply that any retailer has to adopt these clauses. This implication is however far away from the reality; usually, if CCs are offered in some market, then only by a few retailers.

A further strong objection has been put forward by Hviid and Shaffer (1994) and Corts (1995). They argue that even though all retailers have adopted CCs, collusion can still be levered out if one of the retailers turns its CC into the following pledge to its customers: "Whenever my announced price exceeds the lowest price in the market, your are entitled to buy the commodity at a price beating the lowest price by an amount equal to a fixed proportion of the difference between my announced price and the lowest price."

Such beating competition clause (BCC) enables the retailer to undersell the competitors by "overcutting", i.e., by announcing a price above the collusive price. Since (conventional) CCs refer to the announced prices, the price paid at its competitors is not affected by such price announcement and, thus, still corresponds to the collusive price. However, by exercising the BCC of the retailer, its customers pay a lower price. In consequence, the retailer is able to undercut the collusive price without unleashing the CCs of its competitors.

Apart from that, Hviid and Shaffer (1999) have pointed to another factor which could undermine the collusive efficacy of CCs, the hassle costs of the customers. Such costs encompass the time and effort a customer has to spend in order to enforce the CCs (e.g., by providing the required proofs and by finding a qualified salesperson). The consequences of these costs are that CCs like the MCC become a blunt instrument to deter competitors from defection. To understand this argument, let us consider a competitor undercutting the collusive price. The retailers with MCCs immediately match this lower price. However, the effective price (i.e., the price including the hassle) the customers of those retailers would have to pay still exceeds the announced price of the competitor; with the final consequence that these customers finally switch to the defecting competitor.

Also partly in view of these criticisms, competition economists have sought out motivations others than tacit collusion for the use of low price guarantees. Starting with the paper of Png and Hirshleifer (1987), some of them regard CCs as a tool to price differentiate between various groups of consumers, e.g., between the group of uninformed loyals and well informed shoppers. Others like Moorthy and Winter (2006) view such clauses as a device signaling low prices for the customers which are not informed about the actual prices charged by the retailers.

None of these alternative explanations are denied in this paper. However, we question the criticism of the claim that CCs are a device facilitating tacit collusion. Our conjecture is that the points of criticism we invoked above are due to specific market settings assumed in the models. The primary aim of our paper is to investigate this conjecture. In doing so, we will examine the collusive efficacy of CCs in a market setting in several aspects more general than those used in the earlier studies.

Unlike most of those studies, we do not restrict our analysis to the duopoly case. Rather, we consider some homogeneous oligopolistic retail market, i.e., the retailers offer the same commodity and are not spatially differentiated. Moreover, we follow the setup of Corts (1995) and allow retailers to choose any conceivable form of CC - including the CCs most common in real business life.

A further distinctive characteristic of our approach is that we take into account that the retailers compete repeatedly. Since the end point of the competition might be unknown to them, the market is modeled as an infinite multi-stage game. As far as we know, the only two other papers which study the collusive efficacy of CCs within such game-theoretical framework are those of Liu (2013) and Cabral et al. (2021). However, their market settings are significantly more specific than ours. Both articles assume a duopoly and substantially restrict the CC options available for the two retailers. Moreover, they analyze only specific spreading patterns of those CCs so that their results turn out to be incomplete with respect to our concern.

Another peculiarity of our model is that the retailers might have different sales capacities. Most of the previous studies on CCs have (implicitly) taken for granted that the sales capacities of the retailers are unbounded and the demand is equally split among the retailers whenever they announce the same price. Unlike those studies, the retailers can have different market shares in our framework as a result of their heterogeneous sales capacities.

To the best of our knowledge, the only other paper which has so far examined the collusive efficacy of CCs in markets with capacity-constrained retailers is the one of Tumennasan (2013). It rests on the two-stage duopoly framework proposed by Kreps and Scheinkman (1983); the retailers choose their sales capacities in the first stage and fix the commodity prices in the second stage. The novel feature in the model of Tumennasan (2013) is that in the second stage, each duopolist has the additional option to implement the MCC. The results of the paper are mixed. It turned out that the MCC might not necessarily increase the market price in this framework. Indeed, if the capacity costs are sufficiently large, then the MCC have either no effect or even lead to a decrease in the market price.

The approach we pursue deviates substantially from the one of Tumennasan (2013). First, the forms of the CC rather than the sales capacities are endogeneized in our paper; the sales capacities of the retailers are assumed to be invariable for an indefinite period of time and are taken as constant. Besides that, most parts of our analysis do not rely on a specific rationing rule. Instead of assuming efficient rationing as Tumennasan (2013) did, we only require that the rationing rule have some (less contentious) properties which are satisfied by any prominent rationing rule such as the efficient or the proportional rule.[1]

[1] In their seminal paper, Kreps and Scheinkman (1983) have demonstrated that the subgame perfect equilibrium of their two-stage competition game induces exactly the quantities one would obtain from the standard Cournot competition game. This result provides an instructive foundation of Cournot competition. However, as forcefully put forward by Davidson and Deneckere (1986), it proves to be quite fragile. If a rationing rule different from efficient rationing is applied to the game,

This generality enables us to go beyond the fundamental question whether CCs could be used to enforce collusion. If this fundamental claim can be affirmed, our framework allows us to examine the spread and form of the collusive CCs. Regarding the spread, we are in particular interested in the issue whether collusion requires that all retailers adopt CCs or whether it suffices that some of them do it. Regarding the form, it might be interesting to know whether the MCCs induce collusion and, if not, which other forms of CCs could do it.

To address these issues, we have organized the paper as follows. In SECTION 2, we present the market model on which our analysis of competition clause policies is based. The model is an extended capacity-constrained Bertrand competition game; before competing in prices infinitely often, the retailers have to select a binding competition clause policy. This game is solved by the concept of subgame perfectness in SECTION 3. It turns out that if the hassle and implementation costs are sufficiently small and the retailers are neither too short-sighted nor too far-sighted, then partial adoption of CCs suffices to induce collusion.

In SECTION 4, we discuss the existence, robustness, and explicit presentation of the collusive competition clause policies. It is shown that collusion can always be reached by the retailers regardless of which time preferences they have and which rationing rule applies to the residual market demand. However, it is not always possible to specify all collusive competition clause policies without any further assumptions about the underlying rationing rule. We conclude the paper in SECTION 5 with some remarks regarding our approach and possible future research projects. The proofs of our theorems are relegated to the APPENDIX.

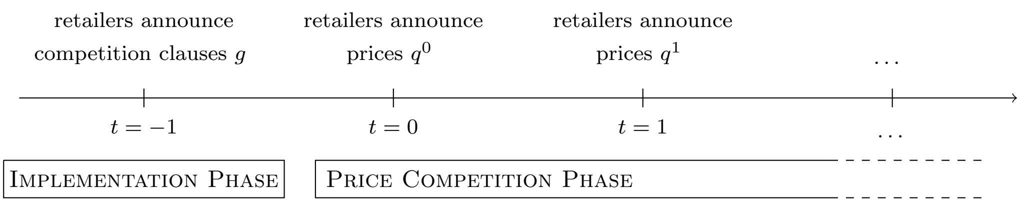
2 Bertrand Markets with Competition Clauses

The markets we study in this paper are oligopolistic commodity markets. Their supply side consists of a set $I := \{1, \dots, n\}$ of retailers where $n \geq 2$. All retailers offer the same commodity and the provision of the commodity generates constant and identical marginal costs $c \geq 0$ for any of them. Moreover, each retailer faces a (positive and real-valued) capacity constraint, the upper limit of the commodities the retailer is able to supply. Without loss of generality, we assume that the retailers' sales capacities differ from each other so that we can arrange them in an increasing order $k_1 < k_2 < \cdots < k_n$.[2] The capacity of the non-empty set $J \subseteq I$ of retailers is summarized by $K_J := \sum_{j \in J} k_j$. We define $K_\emptyset := 0$ and denote the total (market-wide) capacity by $K := K_I$. Retailer i's share of the total capacity is denoted by $\kappa_i := \frac{k_i}{K}$ and the coalition J's share of the total capacity by $\kappa_J := \frac{K_J}{K}$.

The competition between the retailers is modeled as a multi-stage game with infinite horizon. In the first stage (period $t = -1$), the retailers simultaneously announce their competition clause policies. Afterwards, the retailers participate in an infinitely repeated Bertrand competition, i.e., they simultaneously announce prices for the commodity in any of the succeeding and infinitely countable stages (periods $t = 0, 1, \dots$). We call the first stage the *clause implementation phase* and the succeeding stages the *price competition phase*. The timing of our competition game is depicted by the below figure in which the arrow represents the time axis.

e.g., the proportional one, then it cannot any more be upheld. To avoid such fragility of our results, we have abstained from assuming a specific rationing rule. Rather, we have striven to impose only some fundamental and less demanding requirements on the rationing rules.

[2] We remark that our analysis could be extended to include cases in which retailers have identical sales capacities. However, such generalization would expand the set of solutions without any added value. It then would hold: If outcome o satisfies property P, then outcome $\tilde{o}$ which differs from o only by permuting the retailers so that the capacities are still ordered in a non-decreasing way would also satisfy this property. To circumvent such multiplicity, we have refrained from such generalization.



Our game-theoretical setup resembles that adopted in the literature on (partial) cartel formation, e.g. in Selten (1973), D'Aspremont et al. (1983), Escrihuela-Villar (2008), and Bos and Harrington (2010). Those studies endogenize the formation of cartels by multi-stage games where a cartel participation stage in which the firms decide about whether to enter the cartel precedes the competition stage. The competition between the firms is treated differently in those studies; either as a finite game like in the two former articles or as an infinitely repeated game like in the later two articles. Besides the time horizon, our setup also has in common with Bos and Harrington (2010) that the firms are heterogeneous with regard to their sales capacities.

An overview of the game-theoretical studies on competition clause policies reveals that different timing structures regarding the retailers' decisions have been studied. Some authors like Doyle (1988), Corts (1995), and Kaplan (2000) take for granted that each retailer decides simultaneously about the adoption of CCs and the advertised prices. A sequential timing structure in which the competition clauses take the form of binding commitments and are announced before the prices are fixed has been adopted e.g. in Logan and Lutter (1989), Zhang (1995), Chen (1995), and Liu (2013). However, different time horizons of the price competition phase are assumed in these articles. The former three regard price competition as a one-stage game, whereas the latter models it as an infinitely repeated game like we do.

In the following three subsections, we will detail the peculiarities of our market model. This exposition follows the chronological order of the game. First, we specify the options available for the retailers in the clause implementation phase. After that, we turn to the price competition phase and describe the market environment the retailers face. A comprehensive game-theoretical description of our model is given in the last subsection.

To abridge the succeeding presentation, we introduce additional notation. Let A be some set. The indicator mapping of set A is denoted by $\mathbf{1}_A : X \to \mathbb{R}$, i.e., $\mathbf{1}_A(x) = 1$ if $x \in A$ and $\mathbf{1}_A(x) = 0$ otherwise. The cardinality of A is expressed by $|A|$. By definition, $|I| = n$. As is standard, we denote the sets of integers and real numbers by $\mathbb{Z}$ and $\mathbb{R}$, respectively. The set of the non-negative integers and non-negative real numbers are represented by $\mathbb{Z}_+$ and $\mathbb{R}_+$, respectively. If the number zero is excluded from $\mathbb{R}_+$, we write $\mathbb{R}_{++}$. The set of the non-negative real n-tuples is denoted by $\mathbb{R}^I_+$.

Subsets of I are called coalitions of retailers. For any $k \in I$, we define $I_k := \{k, \dots, n\}$, i.e., I_k is the coalition of the $n + 1 - k$ largest retailers. Obviously, $I_1 = I$. Let J be some coalition of retailers. We denote the complement of J by $-J := I \setminus J$. An n-tuple $x := (x_i)_{i \in I}$ is referred to as a *profile of realizations*. We sometimes express profile x by (x_J, x_{-J}) and, if $J = \{i\}$, simply by (x_i, x_{-i}). If all values of profile x are real numbers, then x is termed numerical. The lowest value of a numerical profile x is denoted by $x_{\min} := \min\{x_i : i \in I\}$ and its greatest value by $x_{\max} := \max\{x_i : i \in I\}$.

Suppose some numerical profile $x := (x_i)_{i \in I}$ and some $\alpha \in \mathbb{R}$. We denote the set of all retailers which have realized a value equal to α by $[x = \alpha]$. In general, if R is binary relation on $\mathbb{R}$, we define $[xR\alpha] := \{i \in I : x_i R \alpha\}$. Let σ be a permutation on I so that composition $y := x \circ \sigma$ gives the values of x in a non-decreasing order, i.e., $i < j$ whenever $x_{\sigma(i)} < x_{\sigma(j)}$. We remark that y_i represents the i-th smallest value among the values listed in x. It is called the i-th order statistic of x and is henceforth denoted by $x_{(i)}$. Obviously, $x_{(1)} = x_{\min}$ and $x_{(n)} = x_{\max}$.

2.1 Clause Implementation Phase

The first stage of our competition game is the clause implementation stage. In this phase, each retailer chooses a *competition clause policy* g_i. Such a policy is described as a mapping $g_i : \mathbb{R}_+^I \to \mathbb{R}_+$ which specifies the sales price $g_i(q)$ guaranteed by retailer i for any announcement $q := (q_i)_{i \in I}$ of prices. In accordance with the terminology widely used in the Industrial Organization literature, we call $g_i(q)$ the *guaranteed price* of retailer i and profile q the retailers' *advertised prices.*

From now on, we also take for granted that any competition clause policy g_i satisfies condition

$$\textbf{(G)} \qquad g_i(q) \in \begin{cases} [c, q_i] & \text{if } q_i > \max\{q_{\min}, c\}, \\ \{q_i\} & \text{otherwise.} \end{cases}$$

and denote the set of the competition clauses policies of retailer i satisyfing Assumption (G) by G_i. In the implementation stage, each retailer $i \in I$ is faced with the decision problem to select one of the policies from set G_i.

According to the above definition, the competition clauses policies are tied to the advertised prices, but not to the guaranteed ones. In this regard, we follow the approach of Corts (1995) rather than that of Kaplan (2000). However, the set of available competition clause policies in our setting substantially differs from the one assumed in Corts (1995). While Corts (1995) requires that the price guaranteed by a retailer be based only on two advertised prices, the price advertised by the retailer and the lowest advertised price in the market, we allow that the guaranteed price depends on any advertised price. Our generalization therefore includes CCs directed against some specific competitors.[3]

Moreover, we point out that Assumption (G) imposes several restrictions on the competition clause policies. First, it requires that the guaranteed price correspond to the advertised price whenever the advertised price is the lowest advertised price in the market. This restriction precludes so-called "beat-any-deal" CCs. Such clauses promise the customers to undercut any (and not only any lower) price advertised by some competitor at least by some specified amount or percentage.

Apart from that, Assumption (G) ensures that if the advertised price does not fall short of the marginal costs, then the guaranteed price does not so either. This requirement can be interpreted as an exit option for the retailer. It precludes that a CC-adopting retailer might be forced to sell the commodity at a price below its marginal cost. A justification of this restriction might be that such loss-making situations are not sustainable so that sooner or later such CCs will be abandoned.[4]

The latter restriction is in line with several theoretical studies on CCs which also take for granted that there is a lower bound on the guaranteed prices; for example, Kaplan (2000) assumes that the guaranteed prices are non-negative like any other price. In contrast, Corts (1995) does not impose such restriction. Nevertheless, as can be easily checked, any of the results derived in Corts (1995) are also valid if our setting is adopted. We remark that we could generalize our analysis by assuming

[3] A real life example of such a selective CC is the Amazon and Walmart price match promise of the US online clothing and homeware retailer Zulily. On its website (see `https://www.zulily.com/best-price-promise/41483705?fromEvent=5564`, retrieved at January 13, 2021), Zulily pledges its customers: "If you spot one of our finds at a lower price at Amazon.com or Walmart.com before we do, we'll gladly match their price for the same size and color!"

[4] An illustrative example about how the implementation of a unrestricted CC inflicted losses on the retailer is the incident reported by Robin Young in the article "Tesco Toes the Line in Sock Price War" in The Times on February 3, 1998. UK retailer Tesco was selling a certain brand of sport socks at a package price of £8 and guaranteed to refund its customers by the double of the price difference if they find a competitor offering the product at a lower price. Essential Sports, a small sporting good store, responded to Tesco's price-beating guarantee by selling the same brand of sports socks at a package price of 10p. By invoking the offer of Essential Sports, customers of Tesco received a net payment of £7.80 for acquiring a package of sport socks. Arbatskaya et al. (2004) already pointed to this absurd case in their empirical study on the incidence and variety of competition clauses.

less restrictive lower bounds on the guaranteed prices. However, we decided to refrain from such generalizations as it would make our analysis more tedious without gaining additional insights.

The simplest competition clause policy is the one stipulating that the guaranteed price always corresponds to the advertised price. This policy is defined by $w_i(q) := q_i$ for any profile q of advertised prices and is called the trivial competition clause policy or, simply, the *no clause option*. Due to Assumption (G), any other competition clause policy g_i has the property $c \leq g_i(q) < q_i$ for some profile q of advertised prices satisfying $q_i > \max\{q_{\min}, c\}$. Henceforth, we denote the set containing all non-trivial competition clause policies (i.e., the CCs) available for retailer i by C_i.

A *CC with lump sum refund* entitles the customers to purchase the commodity at a price equal to competitors' lowest advertised price minus a specific fixed amount if the retailer fails to advertise the lowest price in the market. In detail: Let $\mu \in \mathbb{R}$. A competition clause $g_i^{€,\mu}$ of retailer i is said to be a CC with lump sum refund μ if

$$g_i^{€,\mu}(q) := \min\{\max\{q_{\min} - \mathbf{1}_{[q>q_{\min}]}(i)\,\mu, c\}, q_i\}$$

for any profile q of advertised prices.

Obviously, the CC of the form $m_i := g_i^{€,0}$ entitles the customer to purchase the commodity at the lowest advertised price in the market. This clause is referred to as the *meeting competition clause*, also known as the price-matching guarantee. It is the most prominent type of a CC, and most of the scientific literature on CCs focuses on this type. A CC of the form $b_i^{€,\mu} := g_i^{€,\mu}$ with $\mu > 0$ is referred to as a *beating competition clause with a lump sum refund*. It entitles the customers to purchase the commodity at a price falling short of the lowest advertised price in the market by the amount μ if the retailer fails to advertise the lowest price in the market.[5]

A *clause profile* $g := (g_i)_{i\in I}$ summarizes the competition clause policies chosen in the retail market. For example, profile $w := (w_i)_{i\in I}$ describes the situation in which none of the retailers offers a CC and profile $m := (m_i)_{i\in I}$ describes the situation in which all retailers offer the MCC. The set of the clause profiles in which the retailers' competition clause policies satisfy Assumption (G) is denoted by $G := \times_{i\in I} G_i$. Let us pick some arbitrary clause profile $g \in G$. We define $C(g) := \{i \in I : g_i \in C_i\}$ as the set of retailers which have adopted a CC in clause profile g. If there is a $\mu_i \in \mathbb{R}$ for any $i \in C(g)$ so that $g_i = g_i^{€,\mu_i}$, i.e., if each CC-adopting retailer chooses a CC with a lump sum refund, then g is referred to as a *conventional clause profile*.

Our competition model adopts some of the peculiarities of the models of Chen (1995) as well as of Hviid and Shaffer (1999). Like Chen (1995), we assume that implementing CCs causes one-off costs for the retailers. All retailers implementing CCs incur the same fixed costs in the amount of $f > 0$ regardless of the chosen type of CC. The implementation costs encompass the costs of creating the technical and personnel prerequisites for implementing a CC as well as of making the CC publicly known. Since such expenses are largely one-off and more or less the same for any of the CC-adopting retailers, our assumption of fixed and identical implementation costs might be reasonable. Only retailers offering no CCs bear no fixed costs. For the sake of simplification, we take for granted that f is discounted to period 0.

Like Hviid and Shaffer (1999), we assume that exercising CCs might be costly for the customers. All customers making use of a CC incur the same hassle costs in the amount of $z \geq 0$ per purchased unit of the commodity regardless of at which retailer the commodity has been purchased and which type of CC has been offered. The existence of hassle costs might be justified by the real life experience

[5]For example, such BCC is offered by the New Zealand tyre retailer Tony's Tyre Service. They announce on their website (see `https://www.tonystyreservice.co.nz/tyres/price-beat-guarantee`, retrieved on January 13, 2021): "We stock leading tyre brands, at discount prices. Find a lower cash price and we guarantee to beat it by [NZ]$ 10 a tyre on a similar quality product."

that exercising CCs is usually not a smooth process. In general, the burden of proof rests on the customers. They have to spend time and effort to receive the refund guaranteed by the CC, e.g., for providing enough and sound evidence, seeking out qualified salespersons and raising the issue with them. If one takes for granted that each customer buys one unit of the commodity, our assumption that the hassle costs are measured per unit of the commodity seems plausible. The assumption that the hassle costs are identical among the customers has been made for reasons of simplification.

The *effective purchase price* gives the costs the customer incurs for acquiring a unit of the commodity. This price includes the hassle costs whenever the customer has exercised the retailer's CC. The *effective sales price* is the revenue the retailer earns per sold unit of the commodity. To provide a formal specification of these prices, consider the situation in which retailer i has opted for the competition clause policy g_i and the retailers in the market advertise prices $q = (q_j)_{j\in I}$. The effective purchase price of the commodity at retailer i is determined by formula

$$q_i^{\mathrm{p}} := g_i^{\mathrm{p}}(q) := q_i + \mathbf{1}_{C_i}(g_i)\ \min\{g_i(q) + z - q_i, 0\}$$

and the effective sales price by formula

$$q_i^{\mathrm{s}} := g_i^{\mathrm{s}}(q) := q_i - \mathbf{1}_{\mathbb{R}_{++}}(q_i - g_i^{\mathrm{p}}(q))\,(q_i - g_i(q)).$$

Obviously, if $g_i = w_i$ (i.e., retailer i does not adopt a CC) or $g_i(q) + z \geq q_i$ (i.e., it is not worthwhile for the customers to make use of retailer i's CC), then both the effective purchase and the effective sales price at retailer i are equal to the advertised price. Otherwise, the effective sales price corresponds to the price guaranteed by the retailers and the effective purchase price is the effective sales price plus the hassle costs.

2.2 Price Competition Phase

Having adopted their competition clause profiles $g := (g_i)_{i\in I}$, the retailers take part in an infinitely repeated Bertrand competition. At each stage $t \in \mathbb{Z}_+$ of this phase, the retailers simultaneously advertise a price for the commodity. We denote the price advertised by retailer i at stage t by q_i^t and the profile listing all prices advertised at stage t by $q^t := (q_i^t)_{i\in I}$. The effective sales and purchase prices at stage t are then given by $q^{\mathrm{s},t} := g^{\mathrm{s}}(q^t)$ and $q^{\mathrm{p},t} := g^{\mathrm{p}}(q^t)$, respectively.

The demand side at stage t is described by *market demand mapping* $D : \mathbb{R}_+ \to \mathbb{R}_+$. Its value $D(p)$ indicates the total quantity of the commodity demanded by the consumers if they have to pay price p per unit of the commodity. The market demand mapping is time invariant and, therefore, not marked with a stage index t. Moreover, we assume that

(D1) D is continuous,

(D2) there is some $\bar{p} > c$ so that $D^{-1}(0) = [\bar{p}, +\infty[$ and D is decreasing on $[0, \bar{p}[$.

These postulates are standard. Price $\bar{p}$ gives the highest amount the consumers are willing to pay for the commodity and is known as the reservation price of the demand side. Due to Assumptions (D1) and (D2), the *monopolistic profit mapping* $\pi : \mathbb{R}_+ \to \mathbb{R}$ given by $\pi(p) := (p-c)D(p)$ is continuous and positive on open interval $]c, \bar{p}[$. Regarding the form of the profit mapping, it is taken for granted that

(D3) π is strictly quasiconcave on $]c, \bar{p}[$.

It follows from Assumptions (D1), (D2), and (D3) that there exists a unique p^{m} which maximizes π. We term price p^{m} as the *collusive price* and denote the monopolistic profit attained this price by

$\pi^{\mathrm{m}} := \pi(p^{\mathrm{m}})$. One can show that Assumption (D3) results if demand mapping D satisfies (D2) and is concave on $]c, \bar{p}[$.[6] A further requirement we impose on our competition model is that

(D4) $D(c) \leq K_{-i}$ for any $i \in I$.

Assumption (D4) states that the capacities of any coalition of n-1 retailers are sufficient to meet the market demand at price equal marginal costs. If the capacity-constrained retailers were to take part in a static Bertrand competition without CCs, then this assumption would entail that (i) the situation in which each retailer charges a price equal to the marginal costs is a Nash equilibrium and (ii) each retailer earns a zero profit in any Nash equilibrium.

Suppose that the consumers face purchase prices $p := (p_i)_{i \in I}$ at stage t. Moreover, consider some additional and hypothetical purchase price r. The residual market demand at r is defined as the market demand at r not met by the capacities of the retailers undercutting price r. A rationing rule would determine the exact size of residual market demand $R(r|p)$. In the following, we present three rationing rules, the efficient, the proportional and the perfect one. The former two have already been extensively applied in the scientific literature.

- The *efficient rationing rule* $R_{\mathrm{e}}(\cdot|\cdot)$ has been proposed by Levitan and Shubik (1972) as well as Kreps and Scheinkman (1983). It lays down that consumers with the highest willingness to pay are served first. In formal terms, the residual market demand resulting from efficient rationing is given by
$$R_{\mathrm{e}}(r|p) := \max\{D(r) - K_{[p<r]}, 0\}$$
for any profile $p \in \mathbb{R}_+^I$ of purchase prices and any hypothetical purchase price $r \in \mathbb{R}_+$.
- The *proportional rationing rule* $R_{\mathrm{p}}(\cdot|\cdot)$ has been advocated by Beckmann (1965) as well as Davidson and Deneckere (1986). It stipulates that each of the consumers have the same probability of being served. The residual market demand resulting from proportional rationing is inductively specified by
$$R_{\mathrm{p}}(r|p) := \begin{cases} D(r) & \text{for any } r \leq p_{(1)}, \\ \max\left\{\frac{R_{\mathrm{p}}(p_{(i)}|p) - K_{[p=p_{(i)}]}}{D(p_{(i)}|p)}, 0\right\} D(r) & \text{for any } p_{(i)} < r \leq p_{(i+1)} \end{cases}$$
where $p_{(i)}$ is the i-th order statistic of price profile p (i.e., the i-th smallest price in p) and $p_{(n+1)} := +\infty$.
- The *perfect rationing rule* is the opposite extreme of the efficient rationing rule. This rule stipulates that consumers with the lowest willingness to pay are served first. The residual market demand resulting from perfect rationing is inductively specified by
$$R_{\mathrm{l}}(r|p) := \begin{cases} D(r) & \text{for any } r \leq p_{(1)}, \\ \min\left\{D(r), \max\{R_{\mathrm{l}}(p_{(i)}|p) - K_{[p=p_{(i)}]}, 0\}\right\} & \text{for any } p_{(i)} < r \leq p_{(i+1)} \end{cases}$$
where $p_{(i)}$ is the i-th order statistic of price profile p and $p_{(n+1)} := +\infty$.

[6] To see this, we take into account that Assumption (D2) implies $(D(p) - D(p'))\,(p - p') < 0$ for any different prices $p, p' \in]c, \bar{p}[$. Consider some arbitrary, but different prices $p, p' \in]c, \bar{p}[$. Without any loss of generality, we can suppose that $\pi(p') \leq \pi(p)$. It holds:
$$\begin{aligned} \pi(p') &\leq \lambda\pi(p) + (1-\lambda)\pi(p') \\ &= \left(\lambda D(p) + (1-\lambda)D(p')\right)\left(\lambda p + (1-\lambda)p' - c\right) + \lambda(1-\lambda)\left(D(p) - D(p')\right)(p - p') \\ &< D\left(\lambda p + (1-\lambda)p'\right)\left(\lambda p + (1-\lambda)p' - c\right) \\ &= \pi\left(\lambda p + (1-\lambda)p'\right) \end{aligned}$$
for any $\lambda \in]0, 1[$.

In most parts of this paper, we abstain from assuming a specific rationing rule . Rather, we only postulate that *residual market demand mapping* $R : \mathbb{R}_+ \times \mathbb{R}^I_+ \to \mathbb{R}_+$ satisfy the property

(R1) $R_{\mathrm{e}}(r|p) \leq R(r|p) \leq R_{\mathrm{l}}(r|p)$,

(R2) $R(r|\tilde{p}) = R(r|p)$ if $\tilde{p}_i = p_i$ for any $i \in [p < r]$ and $\tilde{p}_i \geq r$ for any $i \in [p \geq r]$.

Assumption (R1) requires that the lower and upper bound of the residual market demand be the residual market demands resulting from the efficient and perfect rationing rule, respectively. Assumption (R2) states that the residual market demand at price r is unaffected by price changes in which the prices below r are kept constant and the other prices do not fall below r. Obviously, both assumptions are innocuous as any reasonable rationing rule is compatible with it; for example, the three rationing rules specified above.

The quantity of the commodity demanded by the consumers from retailer i at stage t and purchase prices $p := (p_i)_{i \in I}$ is derived from the residual market demand. We assume that *demand mapping* $D_i : \mathbb{R}^I_+ \to \mathbb{R}_+$ *of retailer* i is specified according to the allocation rule

(I) $D_i(p) := \dfrac{k_i}{K_{[p=p_i]}} R(p_i|p)$

for any profile $p \in \mathbb{R}^I_+$ of purchase prices.

According to this definition, the proportion of the residual market demand directed to retailer i corresponds to i's share of the total capacity of the retailers charging the same price as retailer i. An argument substantiating such demand allocation is that consumers are more likely to meet retailers with higher sales capacities than those with lower sales capacities so that they more likely to buy the commodity from the former retailers than from the later ones. This allocation rule has already been applied in other studies on Bertrand competition with capacity constraints; e.g., in Allen and Hellwig (1986) as well as Osborne and Pitchik (1986).[7]

A retailer i is able to serve demand $D_i(p)$ as long as the demand does not exceed its capacity constraint k_i. The mapping $X_i : \mathbb{R}^I_+ \to \mathbb{R}_+$ indicating the quantity

$$X_i(p) := \min\{k_i, D_i(p)\}$$

of the commodity retailer i is able to sell at profile p of purchase prices is referred to as the *sales mapping of retailer* i. We conclude from Assumptions (R1) and (I) that if the purchase price of the commodity is the same at any retailer (i.e., $p_i = p_j$ for any $j \in I$), then retailer i's share of the total sales equals κ_i. For this reason, it is justified to interpret κ_i as retailer i's market share at price p. Moreover, Assumption (D4) entails that κ_i corresponds to the proportion of the market demand retailer i serves if $p_i = p_j$ for any $j \in I$ and $p_i \geq c$.

Let us now turn to the situation in which the retailers have implemented competition clause policies g and advertise prices $q := (q_i)_{i \in I}$ in period t. The profit retailer i earns in this period is given by

$$\pi^g_i(q) := (q^{\mathrm{s}}_i - c) X_i(q^{\mathrm{p}}).$$

Recall that $q^{\mathrm{s}}_i := g^{\mathrm{s}}_i(q)$ is the effective sales price charged by retailer i and $q^{\mathrm{p}} := g^{\mathrm{p}}(q)$ summarizes the effective purchase prices in the market. The mapping $\pi^g_i : \mathbb{R}^I_+ \to \mathbb{R}$ specifying retailer i's period profit $\pi^g_i(q)$ for any profile q of advertised prices given that competition clause policies g have been

[7] We remark that not any theoretical analysis of markets with capacity-constrained retailers follows this allocation rule. For example, Kreps and Scheinkman (1983) as well as Davidson and Deneckere (1986) assume that the market demand is equally split among the duopolists whenever they set the same price and each duopolist has a capacity meeting at least the half of the market demand. That is, both retailers have the same market share in this case regardless of their shares of the total capacity. The same holds for the model of Tumennasan (2013), which is based on the framework of Kreps and Scheinkman (1983).

implemented is referred to as *retailer i's profit mapping under clause profile g*. To simplify the notation, we write π_i instead of π_i^w. Obviously, $\pi_i(q) = (q_i - c)X_i(q)$ for any profile q of advertised prices.

The *outcome* of our competition game summarizes all actions which the retailers have chosen during the course of the game. An outcome is also called a *terminal history* and is represented by a sequence $o := (o_t)_{t=-1}^{\infty} := (g, q^0, q^1, \cdots)$ where g indicates the competition clause policy selected in period $t = -1$ and q^t the prices advertised in period t. The retailers are assumed to discount their future profits by a *common discount factor* $0 \leq \delta < 1$ so that the total profit of any retailer i amounts to

$$\Pi_i(o) := \sum_{t=0}^{\infty} \delta^t \pi_i^g(q^t) - \mathbf{1}_{C_i}(g_i) f.$$

Let $O := G \times \left(\times_{t=0}^{\infty} \mathbb{R}_+^I\right)$ be the set of possible game outcomes. The mapping $\Pi_i : O \to \mathbb{R}$ specifying the total profit of retailer i for any game outcome is called the *total profit mapping of retailer i*.

We remark that the common discount factor δ is allowed to take the value of zero. At $\delta = 0$, the retailers do not value future profits. In this case, our competition game transforms itself - in essence - into a two-stage game where the price competition phase consists of only one stage. Obviously, it then holds $\Pi_i(o) := \pi_i^g(q^0)$ for any game outcome $o := (g, q^0, \dots) \in O$. While $\delta > 0$ could be interpreted as a situation in which the end date of the competition is not foreseeable by the retailers, $\delta = 0$ represents the situation in which the end date is commonly known.[8] Our analysis takes into consideration both situations.

An outcome in which each retailer advertises the collusive price p^m in each period of the price competition phase is referred to as a *collusive outcome*. As can be easily checked, if a collusive outcome $o^{\mathrm{m}} := (g, q^0, q^1, \dots)$ is realized, i.e., $q_j^t = p^{\mathrm{m}}$ for any period $t \in \mathbb{Z}_+$ and any retailer $j \in I$, then retailer i earns a total profit in the amount of

$$\Pi_i(o^{\mathrm{m}}) = \frac{1}{1-\delta} \kappa_i \, \pi^{\mathrm{m}} \; - \; \mathbf{1}_{C_i}(g_i) \, f.$$

An outcome in which each retailer advertises a price equal to marginal costs c in each period of the price competition phase is referred to as a *competitive outcome*. Obviously, if a competitive outcome $o^{\mathrm{p}} := (g, q^0, q^1, \dots)$ is realized, i.e., $q_j^t = c$ for any period $t \in \mathbb{Z}_+$ and any retailer $j \in I$, then retailer i earns a total profit in the amount of

$$\Pi_i(o^{\mathrm{p}}) = - \, \mathbf{1}_{C_i}(g_i) \, f,$$

i.e., retailer i incurs a total profit of zero in case it has not adopted a CC, and a loss of f otherwise.

2.3 Business Policies

The rules of our extended capacity-constrained Bertrand competition game we detailed in the preceding subsections are summarized by $\Gamma(\delta, f, n, z)$ or, simply, by Greek capital letter Γ whenever no specific reference is made to the parameters of the game. In the remainder of this section, we introduce formal terms by which we describe the possible courses of our competition game and the strategies available for the retailers. Our notation mainly follows the one proposed in Chapter 6 of the textbook of Osborne and Rubinstein (1994) for multi-stage games with perfect information.

[8] We note that it suffices for our purposes to represent the competition games with finite horizon by two-stage competition games. Resorting to backward induction arguments, one can show that a subgame perfect price policy inducing the collusive price in any of the finitely many stages of the price competition phase exists if, and only if, the collusive price constitutes a Nash equilibrium in the single-stage price competition phase. Therefore, the results obtained in competition games with finite horizon are by nature equal to the ones obtained in the two-stage competition games.

A history up to period t is a sequence enumerating the actions chosen by the retailers until period t. Let h^t be such a history and pick some period $t_0 \leq t$ and some retailer $i \in I$. The component h^t_{i,t_0} of h^t indicates the action retailer i has chosen in period t_0 according to history h^t The n-tuple $h^t_{t_0} := (h^t_{i,t_0})_{i \in I}$ lists the actions of all retailers in period t_0 according to history h^t.

We recursively define $H^{-1} := G$ and $H^t := H^{t-1} \times \mathbb{R}^I_+$ for any $t \in \mathbb{Z}_+$. Apparently, H^t consists of all possible histories up to period t. Moreover, we define singleton $H^{-2} := \{\emptyset\}$ where $\emptyset$ stands for the initial history (starting point) of our competition game. The set of all non-terminal histories is denoted by $H := \cup_{t=-2}^{\infty} H^t$.

Let us choose some history $h \in H^{t_0}$ and pick some period $t \geq t_0$. A history $h^t \in H^t$ is said to be consistent with history h whenever the actions in the periods up to t_0 of history h^t correspond to those of history h, i.e., $h^t_{i,s} = h_{i,s}$ for any $i \in I$ and any $s \leq t_0$. We recursively define $H^{t_0}_h := \{h\}$ and $H^t_h := H^{t-1}_h \times \mathbb{R}^I_+$ for any period $t \geq t_0$. Apparently, H^t_h represents the set of histories up to period t consistent with history h and $H_h := \cup_{t=t_0}^{\infty} H^t_h$ represents the set of all non-terminal histories consistent with history h. To simplify our notation, if $h := (g) \in H^{-1}$, then we simply write H_g instead of $H_{(g)}$.

A strategy or, synonymously, a *business policy of retailer* i is a complete plan of action. It prescribes the actions retailer i takes for any conceivable history. More precisely, it specifies which competition clause policy retailer i selects at the beginning of the game and, for any history $h^{t-1} \in H^{t-1}$ and $t \in \mathbb{Z}_+$, which price retailer i would advertise in period t if he observed the previous actions h^{t-1}. In formal terms, a business policy of retailer i is described as a mapping $s_i : H \to G_i \cup \mathbb{R}_+$ where $s_i(\emptyset) \in G_i$ and $s_i(h^{t-1}) \in \mathbb{R}_+$ for any $h^{t-1} \in H^{t-1}$ and $t \in \mathbb{Z}_+$. The set of business policies available for retailer i is denoted by S_i.

A *business policy profile* $s := (s_i)_{i \in I}$ lists the business policies chosen by all retailers. We denote the set of these profiles by $S := \times_{i \in I} S_i$. The outcome induced by business policy profile s is the infinite sequence of actions realized by retailers pursuing these policies. It is specified by

$$o(s) := (s(\emptyset), s(s(\emptyset)), s(s(s(\emptyset))), \dots) \ .$$

Let us consider some non-initial history $h \in H^t$. The outcome induced by history h and business policy profile s is defined as

$$o^h(s) := (h, s(h), s(s(h)), \dots).$$

It describes the actions recorded by history h for the periods up to t and afterwards the actions the retailers would choose in the succeeding periods if they experienced history h before.

We denote the subgame of Γ starting after history h by Γ^h and the restriction of business policy s_i on H_h by s^h_i. The latter mapping specifies the actions of retailer i only for histories which are consistent with history h. Apparently, the set of these restrictions constitutes the business policy set of retailer i in subgame Γ^h. In line with the above rule of notational simplification, if $h := (g) \in H^{-1}$, we simply write Γ^g and s^g_i instead of $\Gamma^{(g)}$ and $s^{(g)}_i$, respectively. Mapping s^g_i is called the *price policy of retailer* i *in subgame* Γ^g. With slight abuse of notation, we sometimes express business policy s_i of retailer i by $(g_i, (s^{\tilde{g}}_i)_{\tilde{g} \in G})$. Correspondingly, a business policy profile is sometimes expressed by $(g, (s^{\tilde{g}})_{\tilde{g} \in G})$ where $g := (g_i)_{i \in I}$ and $s^{\tilde{g}} := (s^{\tilde{g}}_i)_{i \in I}$.

Retailer i is said to follow a *grim-trigger price policy in subgame* Γ^g whenever its business policy s_i satisfies

$$s^g_i(h^{t-1}) = \begin{cases} p^{\mathrm{m}} & \text{if either } t = 0 \text{ or } h^{t-1}_{i,t_0} = p^{\mathrm{m}} \text{ for any period } 0 < t_0 < t \text{ and any } i \in I, \\ c & \text{otherwise,} \end{cases}$$

for any $h^{t-1} \in H^{t-1}_g$ and any $t \in \mathbb{Z}_+$. The grim-trigger price policy states that the retailer advertises the collusive price p^{m} at the beginning of the price competition phase and continues to advertise this price as long as all retailers have advertised the collusive price in any preceding period. However, if the latter is not satisfied, the retailer advertises the competitive price c. We henceforth denote the grim-trigger price policy of retailer i in subgame Γ^g by t_i^g. Obviously, if the retailers realize clause profile g and adopt grim-trigger price policies $t^g := (t_i^g)_{i \in I}$ in subgame Γ^g, then the collusive outcome $(g, p^{\mathrm{m}}, p^{\mathrm{m}}, \dots)$ results.

Retailer i is said to follow a *competitive price policy in subgame* Γ^g whenever its business policy s_i satisfies

$$s_i^g(h^{t-1}) = c$$

for any $h^{t-1} \in H_g^{t-1}$ and any $t \in \mathbb{Z}_+$. The competitive price policy states that the retailer always advertises a price equal to the marginal costs regardless of the prices advertised in the preceding periods. We henceforth denote the competitive price policy of retailer i in subgame Γ^g by c_i^g. Apparently, if the retailers realize clause profile g and adopt competitive price policies $c^g := (c_i^g)_{i \in I}$ in subgame Γ^g, then the competitive outcome $(g, c, c, \dots)$ results.

3 Perfectly Collusive Clause Profiles

In his seminal work, Salop (1986) pointed out that MCCs can be used as a device facilitating collusive outcomes. This remarkable finding is revisited in our paper where several aspects of commercial activities not considered in Salop (1986) are taken into account. Indeed, our more multifaceted setup allows us to go beyond the fundamental issue raised by Salop (1986) and to study the form and spread of collusive CCs in retail markets.

Compared to the analysis of Salop (1986), the set of CC options available for the retailers is substantially extended in our setting. The MCC is here assumed to be just one of the CCs a retailer is able to implement. This extension is motivated by earlier empirical studies like those of Arbatskaya et al. (2004) and Arbatskaya et al. (2006) which suggest that other forms of CCs than the MCC are also popular among the retailers. Interestingly, our analysis shows that even though the MCC fails to be collusive efficacious, other CCs such as the BCCs with lump sum refunds might not.

Another distinguishing feature is the assumed number of retailers. While Salop (1986) discusses the collusive efficacy of the MCC only for the duopoly case, we do not impose restrictions on the number of the competing retailers, except that we rule out the monopoly case. Such generalization allows us to gain insights about to which extent the CCs have to be implemented in the retail market to render them collusively effective.

One of our main results is that a collusive outcome can be achieved even if just some (or in some cases just one) of the retailers adopt CCs. This proposition is line with the general observation in real business life that not all retailers in the market implement CCs; on this see e.g. the market surveys in Moorthy and Zhang (2006) and Jiang et al. (2017). Nevertheless, it is at odds with most of the earlier theoretical studies on CCs like the ones of Doyle (1988), Logan and Lutter (1989), and Corts (1995) claiming that an all retailers encompassing implementation of CCs is required to unleash the collusive potential of CCs.

A further peculiarity of our competition model with CCs is that the retailers are assumed to be heterogeneous with regard to their sales capacities. As will be argued later, such heterogeneity might induce specific patterns in the spread of CCs. To the best of our knowledge, the paper of Tumennasan (2013) and our paper are so far the only ones examining the relationship between capacity size and competition clause policy.

The following analysis of the retailers' business policies is based on the solution concept of subgame perfectness. Since retailers are assumed to interact infinitely often, non-compliant behavior can be punished by retaliatory measures of the competitors in the future. To simplify our analysis, we focus only on those subgame perfect business policies using grim-trigger price policies in the price competition phases whenever possible and competitive price policies otherwise.

3.1 Definition and Alternative Characterizations

The set of the subgame perfect equilibria in our competition game Γ is denoted by $\mathcal{S}(\Gamma)$. As just mentioned, we narrow down the solution set by imposing the restriction

$$\mathcal{S}^{\mathrm{g}}(\Gamma) := \left\{ s \in \mathcal{S}(\Gamma) : s^g = \left\{ \begin{array}{ll} t^g & \text{if } t^g \text{ is a subgame perfect equilibrium in } \Gamma^g, \\ c^g & \text{otherwise} \end{array} \right. \right\}.$$

The business policies of solution set $\mathcal{S}^{\mathrm{g}}(\Gamma)$ have the characteristic that grim-trigger price policies are implemented in a price competition phase whenever they prove to be subgame perfect in this subgame. Otherwise, the retailers pursue the competitive price policy. Obviously, the grim-trigger price policies unleash the most severe punishment for defecting from the collusion.

Although restricting the set of solutions to $\mathcal{S}^{\mathrm{g}}$ is a substantial simplification, it is by far not a peculiarity of our analysis. Such simplification has been made in the theory of partial cartels, like in the models of Escrihuela-Villar (2008) as well as Bos and Harrington (2010) to name a few. Moreover, it has also been proposed by Liu (2013) for the analysis of competition clause policies. Notably, this restriction does not cause existence problems; without difficulty, one can show that solution set $\mathcal{S}^{\mathrm{g}}(\Gamma)$ is non-empty for any competition game Γ.

As the scope of our analysis is restricted to those business policies, we exclusively reserve the term subgame perfectness to business policy profiles belonging to $\mathcal{S}^{\mathrm{g}}(\Gamma)$. A clause profile $\hat{g} := (\hat{g}_i)_{i\in I}$ is said to be *subgame perfect* in Γ, if there exists a subgame perfect business policy profile $\hat{s} := (\hat{g}, (s^g)_{g\in G})$ in Γ, i.e., $\hat{s} \in \mathcal{S}^{\mathrm{g}}(\Gamma)$. As will be argued later, to characterize solution set $\mathcal{S}^{\mathrm{g}}(\Gamma)$, we need to calculate the critical discount factor for any subgame Γ^g.

The *critical discount factor of retailer* i at clause profile g is defined as

$$\delta^g_{i,\mathrm{crit}} := \inf \Delta^g_i \quad \text{where} \quad \Delta^g_i := \left\{ \delta \in [0,1[\, : \frac{1}{1-\delta}\, \kappa_i\, \pi^{\mathrm{m}} \geq \sup_{q_i \neq p_m} \pi^g_i(q_i, p^{\mathrm{m}}_{-i}) \right\}.$$

In words, the critical discount factor $\delta^g_{i,\mathrm{crit}}$ of retailer i at clause profile g corresponds to the lowest discount factor for which the collusive profit of retailer i does not fall short of the highest one-stage profit this retailer can attain by defecting given that clause profile g has been adopted and all competitors of retailer i advertise collusive price p^{m}.

The *critical discount factor at clause profile* g is defined as the maximum individual critical discount factor at this profile, i.e.,

$$\delta^g_{\mathrm{crit}} := \max\{\delta^g_{i,\mathrm{crit}} : i \in I\}.$$

In accordance with the previous notational simplifications, we omit the superscript of the critical discount factors if $g = w$. The following remark summarizes some useful characteristics of the maximum one-stage profit retailer i can attain by defecting from the collusive behavior.

Remark 1. *Consider* $\Gamma(\delta, f, n, z)$*. It holds:*

(a)

$$0 \leq \sup_{c < q_i \neq p^m} \pi^g_i(q_i, p^m_{-i}) = \sup_{q_i \neq p^m} \pi^g_i(q_i, p^m_{-i})$$

for any retailer $i \in I$ *and any clause profile* $g \in G$*.*

(b)

$$\sup_{q_i \neq p^m} \pi^g_i(q_i, p^m_{-i}) \leq (p^m - c) \max\{k_i, D(p^m)\}$$

for any retailer $i \in I$ *and any clause profile* $g \in G$*.*

The two parts of this remark substantially simplify the calculation of the maximum one-stage profit attainable by defection. Parts (a) and (b) provide a lower and an upper bound for this profit, respectively. Moreover, according to part (a), it suffices for the calculation to consider only the advertised prices above the marginal costs.

It turns out that if none of the retailers have adopted a CC (i.e., $g = w$) in the implementation phase, the critical discount factor for retailer i is given as follows.

Remark 2. *Consider* $\Gamma(\delta, f, n, z)$. *It holds*

$$\delta_{i,crit} = 1 - \max\left\{\kappa_i, \tfrac{D(p^m)}{K}\right\} \geq \delta^g_{i,crit}$$

for any retailer $i \in I$ *and any clause profile* $g \in G$.

An immediate consequence of the last remark is that

$$\delta_{\rm crit} = 1 - \max\left\{\kappa_1, \tfrac{D(p^{\rm m})}{K}\right\} \geq \delta^g_{\rm crit}$$

for any clause profile $g \in G$. This means, the critical discount factor for clause profile w is the greatest among the critical discount factors; or putting differently, the adoption of CCs never increases the critical discount factor.

As can be easily checked, our competition game Γ is a multi-stage game being continuous at infinity. We know from Theorem 3.2 in the textbook of Fudenberg and Tirole (1991) that the One-Shot Deviation Principle holds for such games. This principle provides a simple characterization of subgame perfect equilibria. It states that a strategy profile is subgame perfect if, and only if, there exists no profitable one-shot deviation for each subgame and each player.[9] Applying this principle to competition game Γ, we obtain the following characterization of solution concept $\mathcal{S}^{\rm g}$.

Remark 3. *Consider* $\Gamma(\delta, f, n, z)$. *It holds* $\hat{s} := (\hat{g}, (\hat{s}^g)_{g\in G}) \in \mathcal{S}^{\rm g}(\delta, f, n, z)$ *if, and only if, the properties*

(T1) $\hat{s}^g_i = \begin{cases} t^g_i & \text{if } \delta \geq \delta^g_{crit} \\ c^g_i & \text{otherwise} \end{cases}$ *for any* $i \in I$ *and* $g \in G$,

(T2) $\Pi_i(o(\hat{s}) \geq \Pi_i(o(\tilde{s}))$ *for any* $\tilde{s} := (\tilde{s}_i, \hat{s}_{-i}) \in S$ *where* $\tilde{s}_i := (\tilde{g}_i, (\hat{s}^g_i)_{g\in G})$ *and any* $i \in I$

are satisfied.

The focus of our interest is whether solution set $\mathcal{S}^{\rm g}(\Gamma)$ contains business policy profiles inducing collusive price outcomes. We call such profiles perfectly collusive and, henceforth, denote this subset of $\mathcal{S}^{\rm g}(\Gamma)$ by

$$\mathcal{S}^{\rm m}(\Gamma) := \{s \in \mathcal{S}^{\rm g}(\Gamma) : \ o(s) \text{ is a collusive price outcome}\}.$$

A clause profile $\hat{g} := (\hat{g}_i)_{i\in I}$ is said to be *perfectly collusive* in Γ whenever there exists some business policy profile $\hat{s} := (\hat{g}, (\hat{s}^g)_{g\in G}) \in \mathcal{S}^{\rm m}(\Gamma)$.

As will be shown next, such clause profiles can be characterized by three conditions; two refer only to the common discount factor and the remaining one also to the implementation costs. With regard to the latter condition, it turns out to be helpful to introduce the notion of the *critical implementation costs of clause profile* g *at common discount factor* δ, which are defined as

$$f^{g,\delta}_{\rm crit} := \begin{cases} \frac{1}{1-\delta}\kappa_i \pi^{\rm m} & \text{if } C(g) \neq \emptyset \text{ where } i := \min C(g), \\ +\infty & \text{otherwise.} \end{cases}$$

This value gives the collusive profit of the smallest CC-adopting retailer. Obviously, whenever the actual implementation costs exceed $f^{g,\delta}_{\rm crit}$, then this retailer would be better off at the competitive outcome without implementing a CC than at the collusive outcome with implementing a CC.

[9]For further details of this principle, we refer to the proof of Remark 3.

Remark 4. *Consider* $\Gamma(\delta, f, n, z)$. *A clause profile* $\hat{g}$ *is perfectly collusive if, and only if, properties*

(M1) $\delta^{\hat{g}}_{crit} \leq \delta$

(M2) $\delta < \delta^{g}_{crit}$ *for any* $g := (w_i, \hat{g}_{-i}) \in G$ *and any* $i \in C(g)$,

(M3) $f \leq f^{\hat{g},\delta}_{crit}$

are satisfied

The three properties of Remark 4 completely characterize the perfectly collusive clause profiles and correspond to the conditions of external and internal stability in the partial cartel theory, see D'Aspremont et al. (1983). Property (M1) covers the external stability of the collusive clause profile: None of the non CC-adopting retailers has an incentive to adopt a CC as the collusive outcome is already reached and, thus, the implementation of a CC would turn out to be a costly business operation without any additional gain for them. Properties (M2) and (M3) ensure the internal stability of the collusive clause profile: None of the CC-adopting retailers prefers repealing the CC. Such withdrawal would induce a competitive outcome; with the consequence that none of them would be better off.

In the remainder of this section, we discuss whether a collusive price outcome can be achieved for any arbitrary common discount factor and, if so, which kinds of conventional clause profiles could sustain such outcome. It turns out that the characterization of perfectly collusive clause profiles put forward in Remark 4 becomes very useful for tackling these issues.

3.2 Collusion without Competition Clauses

This subsection provides a first insight into the incentives of the retailers to adopt CCs. As will be demonstrated next, the adoption of CCs proves to be a sufficient, but not a necessary condition for inducing collusion.

Proposition 5. *Consider* $\Gamma(\delta, f, n, z)$. *There exists no subgame perfect business policy profile in which a retailer adopts a CC, but which induces the competitive outcome.*

According to this proposition, the only purpose of CCs in our competition model is achieving a collusive outcome. This result relies on our implicit assumption that all consumers know all advertised prices and competition clause policies. If this assumption is dropped, CCs could be adopted for other reasons like signaling low prices or price differentiating between customers with low and high costs in searching out the prices charged by other retailers.[10]

Jain and Srivastava (2000), Moorthy and Winter (2006), as well as Moorthy and Zhang (2006) have strongly put forward the former purpose. Earlier studies advocating the later purpose are the ones of Png and Hirshleifer (1987), Corts (1997), Chen et al. (2001), and Hviid and Shaffer (2012). In these studies, some exogenously determined groups of consumers are able to take advantage of CCs while others are not. More recent studies like the ones of Janssen and Parakhonyak (2013), Yankelevich and Vaughan (2016), and Jiang et al. (2017) have endogenized the price search of the consumers to motivate CCs as a device enabling price discrimination.

Interestingly, as parts of this literature suggest, if CCs are used as a signaling or price discriminating device, then CCs could have pro-competitive and welfare-enhancing effects, reducing (some of) the effective sales prices and making better off not only the retailers but also (at least some groups of) consumers.[11] This finding however is in contrast to our Proposition 5, which rules out any other purpose than enforcing collusion.

[10]Other reasons why retailers adopt CCs could be mentioned. Hviid (2010) provides a comprehensive overview of the different motivations for CCs discussed in the scientific literature.

[11]An opposite point of view has been taken e.g. by Budzinski and Kretschmer (2011). They claim that CCs still induce the collusive price even though there is asymmetric information among the consumers about the price policies of the retailers.

Moreover, we remark that the converse of PROPOSITION 5 does not hold. It turns out that if the common discount factor is sufficiently large, the collusive outcome is achieved only without CCs.

Theorem 6. *Consider $\Gamma(\delta, f, n, z)$. It holds:*

(a) *If clause profile w (i.e. the clause profile in which none of the retailers adopts a CC) is perfectly collusive, then $\delta_{crit} \leq \delta < 1$.*

(b) *If $\delta_{crit} \leq \delta < 1$, then clause profile w is the only perfectly collusive clause profile.*

The PERFECT FOLK THEOREM put forward by Friedman (1971) states that player can reach collusive outcomes in infinitely repeated games whenever the common discount factor is sufficiently large. According to THEOREM 6, this fundamental result remains valid even for our extended Bertrand competition game where CCs can be adopted by the retailers at the beginning. This is due to the fact that if the retailers value future profits sufficiently high, then the threats implicit in the grim-trigger price policies are efficacious and deter the retailers from undercutting monopoly price p^{m}. In this case, adopting CCs causes only undue costs.

However, as can be easily concluded from REMARKS 2 and 3, the retailers would choose the competitive price policy in the price competition phase if the common discount factor δ were below threshold δ_{crit} and none of them had adopted a CC. For such discount factors, the mere grim-trigger threats are too weak to sustain collusion. Whether collusion can then be facilitated through the implementation of CCs, is studied in the remaining parts of this section.

3.3 Collusive Competition Clauses with Lump Sum Refunds

A strong argument against the collusive efficacy of CCs has been put forward by Hviid and Shaffer (1994) and Corts (1995). They show that CCs are collusively ineffective if the competition between the retailers is described as a one-stage game in which the retailers decide simultaneously about the competition clause policies and the advertised prices.[12] However, the robustness of their results might be questionable. It turns out that their conclusion cannot easily transferred to our setting as our modeling of the competition differs from those in two core aspects.

First, the timing of the decision-making in our framework is different. We consider competition clause policies as binding commitments and therefore describe the competition between retailers as a sequential-move game in which before fixing the prices, the retailers announce their competition clause policies. That means, when the retailers advertise the prices, they have already been informed about the competition clause policies pursued by their competitors. Second, our setting allows for a longer time horizon. We take into account that retailers might compete with each other repeatedly (i.e., finitely often until a unknown end point) so that non-cooperative behavior of a retailer can be punished by non-cooperative behavior of its competitors in future periods.

As will be demonstrated below, these divergences from the settings of Hviid and Shaffer (1994) and Corts (1995) are responsible for a different answer regarding the efficacy of CCs. More specifically, for sufficiently small hassle and implementation costs, clause profiles in which a sufficiently large coalition of retailers adopt CCs with well-proportionate lump sum refunds prove to be perfectly collusive in our competition model.

[12]See PROPOSITION 2 of Hviid and Shaffer (1994) and PROPOSITION 2 of Corts (1995). It should be noted that the scope of their results are different. Hviid and Shaffer (1994) prove the claim for any heterogeneous Bertrand duopoly in which the MCC and the BCCs with refund factors on the price difference ("We promise that if we do not offer the lowest price in the market, we'll beat it by an amount equal to x % of the difference between our announced price and the lowest price in the market.") are the only CCs available for the retailers. Corts (1995) proves the claim for any homogeneous Bertrand oligopoly in which the CCs can take on a wide variety of forms including the conventional ones.

One reason why CCs nevertheless might fail to facilitate collusion even in our setting is that the total capacity of the CC-adopting retailers is too small in order to punish defectors. Indeed, if the total capacity of those retailers does not meet the market demand at the monopoly price, each of them faces the same incentive to defect from the collusion as if they had not adopted CCs. The subsequent remark sets this point our and states that the critical discount factors of such clause profiles correspond to the one of the trivial clause profile.

Remark 7. *Consider* $\Gamma(\delta, f, n, z)$*. It holds*

$$\delta_{crit}^{g} = \delta_{crit}$$

for any clause profile g *satisfying* $K_{C(g)} \leq D(p^m)$*.*

An immediate consequence of this remark is that adopting a CC is pointless whenever the total capacity of the CC-adopting retailers does not exceed the market demand at the monopoly price. We summarize this finding in the following theorem.

Theorem 8. *Consider* $\Gamma(\delta, f, n, z)$ *where* $\delta < \delta_{crit}$*. Any clause profile* g *satisfying* $K_{C(g)} \leq D(p^m)$ *is not perfectly collusive.*

Another well-known reason why CCs become collusively ineffective are hassle costs. Such costs drive a wedge between the effective purchase and sales prices whenever the CC is exercised. This wedge might entail that a defecting retailer can undersell any of its competitors, including the ones which have adopted a CC. Even if the sales prices at those competitors are below the reduced price advertised by the defector, the purchase prices at them might exceed this advertised price due to the hassle costs. In such a case, the CC-adopting retailers are unable to immediately penalize defections from collusive behavior, with the consequence that the CCs become collusively ineffective.

Apart from that, even though collusive behavior can be enforced by CCs, retailers nevertheless might abstain from offering them if their implementation is too expensive. Obviously, if the implementation costs exceed the additional profit gained by acting collusively instead of acting competitively, then the retailer would be better off without CC.

In the following, we aim to be more specific about the consequences of the hassle and implementation costs on the collusive efficacy of CCs. In particular, we seek to provide thresholds of the hassle and implementation costs above which collusion is not enforceable by CCs. For this purpose, additional notation is required.

To specify the threshold for the hassle costs, we define mapping $\bar{\pi}_i(\cdot)$ where

$$\bar{\pi}_i(p) := (p - c) \min\{k_i, D(p)\}$$

for any $p \in \mathbb{R}_+$ and any retailer $i \in I$. Apparently, mapping $\bar{\pi}_i(\cdot)$ gives the profit earned by retailer i if the customers of retailer i pay a price p and the customers of the other retailers a price above p. Henceforth, we refer to mapping $\bar{\pi}_i$ as the *monopolistic profit mapping of retailer* i. We are interested in the price $\bar{p}_i^{\delta}$ for which the monopolistic profit of retailer i corresponds to the total (net) surplus retailer i attains at the collusive outcome.

Remark 9. *Consider* $\Gamma(\delta, f, n, z)$*. For any retailer* $i \in I$ *and any common discount factor* $\delta < \delta_{i,crit}$*, there is a unique* $\bar{p}_i^{\delta}$ *so that* $c < \bar{p}_i^{\delta} < p^m$ *and* $\bar{\pi}_i(\bar{p}_i^{\delta}) = \frac{1}{1-\delta} \kappa_i \pi^m$*. Moreover, the conditions*

(i) $\bar{p}_i^{\beta} < \bar{p}_i^{\delta}$

(ii) $\bar{p}_j^{\delta} \leq \bar{p}_i^{\delta}$

are satisfied for any common discount factors $\beta < \delta$ *and any retailers* $j \leq i$*.*

Based on price $\bar{p}_i^\delta$, we define

$$\bar{\mu}_i^\delta := p^{\rm m} - \bar{p}_i^\delta\,, \qquad\qquad \bar{\phi}_i^\delta := \tfrac{p^{\rm m}-\bar{p}_i^\delta}{p^{\rm m}}\,, \qquad\qquad \bar{z}_i^\delta := \bar{p}_i^\delta - c$$

for any $\delta < \delta_{i,\text{crit}}$ and any $i \in I$. Due to REMARK 9, these values are positive. Moreover, this remark implies that $\bar{\mu}_i^\delta$ and $\bar{\phi}_i^\delta$ is decreasing in δ and non-increasing in i, whereas $\bar{z}_i^\delta$ is increasing in δ and non-decreasing in i.

To specify the threshold for the implementation costs, we consider the greatest number $j \in I$ so that the coalition consisting of the $n+1-j$ largest retailers has a total capacity meeting the demand at the monopoly price $p^{\rm m}$; or expressed formally, $j := \max\{k \in I : K_{I_k} \geq D(p^{\rm m})\}$. We define

$$\bar{f}^\delta := \tfrac{1}{1-\delta}\kappa_j \pi^{\rm m}.$$

for any common discount factor $\delta < \delta_{\text{crit}}$. Apparently, $\bar{f}^\delta$ is the total net surplus the retailer with the lowest capacity in coalition I_k achieves at the collusive price outcome. We hint that ASSUMPTION (D4) ensures $\bar{f}^\delta \geq \frac{1}{1-\delta}\kappa_2\pi^{\rm m}$.

The following theorem provides thresholds for the hassle and implementation costs above which CCs become an inefficacious device for collusion.

Theorem 10. *Consider $\Gamma(\delta, f, n, z)$ where $\delta < \delta_{crit}$. If $f > \bar{f}^\delta$ or $z > \bar{z}_1^\delta$, then there is no perfectly collusive clause profile.*

THEOREMS 8 and 10 point towards two technical issues preventing CCs to induce collusive behavior, a too small total capacity of the CC adopting retailers and too large hassle or implementation costs. It remains to examine the collusive efficacy of CCs in cases these technical barriers do not prevail. We address this issue by focusing on the conventional clause profile, i.e., in which the retailers adopt CCs with lump sum refunds.

Our first finding is that CCs with lump sum refunds less than the hassle costs are avoided by the retailers in any circumstance. The reason for this obvious. Since such CCs do not cause a matching or an undercutting of the defector's price reduction, they do not remove the incentive of the retailers to deviate from collusive behavior. Hence, such CCs prove to be ineffective in punishing defectors.

Theorem 11. *Consider $\Gamma(\delta, f, n, z)$. There exists no perfectly collusive clause profile in which a retailer adopts a CC with lump sum refund $\mu < z$.*

One interesting aspect of the last theorem is that it also reproduces the inefficacy result of Hviid and Shaffer (1999) for the case of perfectly homogeneous commodities. As stated in the above theorem, if hassle costs exists, MCCs turn out to be ineffective in facilitating collusion no matter how small these costs are. Due to this disruptive effect, hassle costs have been referred to as the Achilles' heel of the MCCs by Hviid and Shaffer (1999).

This pro-competitive effect of hassle costs has also been experimentally tested by Dugar and Sorensen (2006). Their setting essentially corresponds to ours with common discount factor $\delta = 0$. An augmented homogeneous Bertrand trioloply is played where the three players decide in the first stage whether they adopt the MCC and set the price of the commodity in the second stage. It is assumed that (at least some of) the customers incur hassle costs in exercising the MCC. Dugar and Sorensen (2006) report that if exercising the MCC is tedious for any customer, then the average effective price is close to the one resulting from the games without the MCC option.

We remark that the ineffectiveness result of THEOREM 11 hinges crucially on the assumptions of perfectly homogeneous commodity markets and simultaneous price-setting. If these assumptions are dropped, then it might be the case that some of the CCs mentioned in the theorem become collusively efficacious. Indeed, based on the Hotelling's linear city model with sequential price-setting, it is shown by Trost (2016) that MCCs prove to be collusively efficacious even if hassle costs prevail and by Pollak

(2017) that CCs with negative lump sum refunds (so-called CCs with markups) prove to be collusively efficacious in the case that no hassle costs prevail.

Let us now consider clause profiles in which a sufficiently large coalition of retailers adopts CCs with lump sum refunds in the amount of the hassle costs. Lower and upper bounds of their critical discount factors are presented in the following remark.

Remark 12. *Consider $\Gamma(\delta, f, n, z)$ where $\delta < \delta_{crit}$ and $z \leq \bar{z}_1^{\delta}$. It holds:*

(a) *If clause profile $g := (b_J^{€,z}, w_{-J})$ satisfies $K_J > D(p^m)$, then*

$$\delta_{crit}^{g} \geq 1 - \kappa_J$$

where equality holds if $z \leq \bar{z}_1^{1-\kappa_J}$.

(b) *If clause profile $g := (b_J^{€,z}, w_{-J})$ satisfies $\kappa_J \geq 1 - \delta$, then*

$$\delta_{crit}^{g} \leq \delta$$

where equality holds if, and only if, $\kappa_J = 1 - \delta$ or $z = \bar{z}_1^{\delta}$.

Applying this remark, we conclude that such clause profiles facilitate collusion for discount values below threshold δ_{crit} even though ("mild") implementation and hassle costs exist.

Theorem 13. *Consider $\Gamma(\delta, f, n, z)$ where $\delta < \delta_{crit}$ and $z \leq \bar{z}_1^{\delta}$. The non-trivial clause profile $\hat{g} := (g_J^{€,z}, w_{-J})$ (i.e., the clause profile in which the retailers of non-empty coalition $J \subseteq I$ adopt CCs with lump sum refund z while the other retailers do not adopt CCs) is perfectly collusive if and only if the conditions*

(i) $1 - \kappa_J \leq \delta,$

(ii) $\delta < 1 - \kappa_{J\setminus\{j\}}$ *where* $j := \min J,$

(iii) $f \leq f_{crit}^{\hat{g},\delta}$

are satisfied.

The conditions of this theorem are easy to interpret. Condition (i) represents the external stability condition of the collusive clause profiles. It states that the sum of the market shares of the non CC-adopting retailers has to be lower or equal to the common discount factor. Conditions (ii) and (iii) represent the internal stability condition of the collusive clause profiles. The former states that the sum of the market shares of the non CC-adopting retailers has to be large enough so that it would exceed the common discount factor if the market share of one of CC-adopting retailers were added.

The anti-competitive prediction of Theorem 13 has already been experimentally tested by Dugar (2007) for the specific case in which the price competition phase consists only of one stage (i.e., $\delta = 0$) and no hassle costs exist (i.e., $z = 0$). The benchmark case of this experiment was a standard homogeneous Bertrand triopoly game and the treatment case the two-stage game in which the three players had the option to implement the MCC before competing in the Bertrand game. In line with the prediction of the theorem, it was observed that all three players adopted the MCC in the majority of the treatment cases and the average effective price in the treatment cases was significantly higher than in the benchmark cases.

The essential point of Theorem 13 is that it challenges two fundamental claims made in several theoretical studies on collusive CCs. First, in the case that retailers do not take into account future profits (i.e., $\delta = 0$), we infer from this theorem that the collusive price outcome is realized if all retailers adopt CCs with lump sum refund z. However, this results is at odds with Proposition 2 in Corts (1995) according to which no business policy profile can substantiate supracompetitive prices. The reason for this divergence is that competition clause policies and prices are advertised simultaneously in the model of Corts (1995), whereas in our setting the competition clause policies are considered as binding commitments and are set before the prices are advertised.[13]

Second, if $\delta > 0$, collusive behavior might occur even though only a few retailers adopt CCs with lump sum refund z. In particular, in the absence of hassle costs, a partial adoption of MCCs might suffice to induce collusive behavior. In contrast to our result, Proposition 1.C in Doyle (1988), Theorem 1 in Logan and Lutter (1989), as well as Claim 2 in Corts (1995) state that a necessary condition for making MCCs collusively efficacious is that they have to be adopted by all retailers.

Only few research papers like the ones of Belton (1987), Hviid and Shaffer (1999) as well as Hviid and Shaffer (2010) agree with ours that partial adoption of CCs might be consistent with collusive intentions. However, those papers take for granted some market asymmetries or extended forms of competition clause policies. Belton (1987) assumes sequential price-setting; the retailers not adopting a CC advertise the price after the CC-adopting retailers have advertised theirs. The crucial assumption of the model of Hviid and Shaffer (1999) is that the retailers are spatially differentiated and face strong asymmetric cost or demand conditions. Hviid and Shaffer (2010) consider competition clause policies which combine the MCC with the retroactive most-favored customer clause (MFC).

A merit of our analysis is that one has not to invoke such market asymmetries or additional provisions in the competition clauses to reconcile the theory of CCs with the striking empirical finding that in most commodity markets only a few retailers adopt CCs. We conclude this section by providing a numerical market example illustrating this salient feature of our competition model.

3.4 A Market Example

Let us assume throughout the remaining part of this section that the retail market of our competition model can be described as follows.

Example. *The retail market is a Bertrand market whose demand side is summarized by market demand mapping $D(p) := \max\{1 - p, 0\}$. The residual market demand results from a rationing rule satisfying* Assumptions *(R1) and (R2). The supply side consists of four retailers whose capacities are $k_1 := \frac{2}{10}$, $k_2 := \frac{3}{10}$, $k_3 := \frac{9}{10}$, and $k_4 := \frac{11}{10}$. Each retailer produces at marginal costs of zero (i.e., $c = 0$) and is faced with a demand resulting from* Assumption *(I). The common discount factor of the retailers is $0 \le \delta < 1$ and the implementation costs for CCs are $0 < f \le \frac{1}{1-\delta}\frac{1}{50}$. The customers incur hassle costs $0 \le z \le \frac{1}{10}$ if they exercise CCs.*

Apparently, the market demand mapping of this example satisfies our Assumptions (D1) - (D4). The monopoly price is equal to $p^{\mathrm{m}} = \frac{1}{2}$ so that the monopoly profit corresponds to $\pi^{\mathrm{m}} = \frac{1}{4}$. The market capacity amounts to $K = \frac{25}{10}$ and the market shares of the retailers are $\kappa_1 = \frac{8}{100}$, $\kappa_2 = \frac{12}{100}$, $\kappa_3 = \frac{36}{100}$, and $\kappa_4 = \frac{44}{100}$. As can be easily checked, it holds $f \le \frac{1}{1-\delta}\kappa_1\pi^{\mathrm{m}}$ and $z \le \bar{z}_1^0 = \frac{1}{10}$.

Table I displays the critical discount factors as well as the critical implementation costs of all perfectly collusive clause profiles in which each of the CC-adopting retailers chooses the CC with the lump sum refund equal to the hassle costs. The critical discount factor of the trivial clause profile is calculated according to the formula presented in Remark 2. Since $z \le \bar{z}_1^0$, we can make use of the formula presented in Remark 12(a) in order to specify the critical discount factors of the other clause profiles.

In compliance with Theorem 6, collusion in the market is achieved without adopting CCs whenever $\delta \ge \delta_{\mathrm{crit}} = \frac{80}{100}$. Interestingly, as set forth in Theorem 13, collusion can be sustained even though $\delta < \frac{80}{100}$. However, for such common discount factors, at least some of the retailers have

[13] We note that Proposition 2 in Corts (1995) is derived from a set of available competition clause policies slightly different from ours. However, one can easily show that the same conclusion results if our set G is assumed.

Cardinality of $C(g)$	Clause profile g	Critical discount factor δ^{g}_{crit}	Critical implementation costs $f^{g,\delta}_{\text{crit}}$
0	$(\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}, \mathbf{w_4})$	$\mathbf{\frac{80}{100}}$	
1	$(\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}, \mathbf{g_4^{\text{€},z}})$	$\mathbf{\frac{56}{100}}$	$\mathbf{\frac{1}{1-\delta} \frac{11}{100}}$
	$(w_1, w_2, g_3^{\text{€},z}, w_4)$	$\frac{64}{100}$	$\frac{1}{1-\delta} \frac{9}{100}$
2	$(\mathbf{w_1}, \mathbf{w_2}, \mathbf{g_3^{\text{€},z}}, \mathbf{g_4^{\text{€},z}})$	$\mathbf{\frac{20}{100}}$	$\mathbf{\frac{1}{1-\delta} \frac{9}{100}}$
	$(w_1, g_2^{\text{€},z}, w_3, g_4^{\text{€},z})$	$\frac{44}{100}$	$\frac{1}{1-\delta} \frac{3}{100}$
	$(g_1^{\text{€},z}, w_2, w_3, g_4^{\text{€},z})$	$\frac{48}{100}$	$\frac{1}{1-\delta} \frac{2}{100}$
	$(w_1, g_2^{\text{€},z}, g_3^{\text{€},z}, w_4)$	$\frac{52}{100}$	$\frac{1}{1-\delta} \frac{3}{100}$
	$(g_1^{\text{€},z}, w_2, g_3^{\text{€},z}, w_4)$	$\frac{56}{100}$	$\frac{1}{1-\delta} \frac{2}{100}$
3	$(\mathbf{w_1}, \mathbf{g_2^{\text{€},z}}, \mathbf{g_3^{\text{€},z}}, \mathbf{g_4^{\text{€},z}})$	$\mathbf{\frac{8}{100}}$	$\mathbf{\frac{1}{1-\delta} \frac{3}{100}}$
	$(g_1^{\text{€},z}, w_2, g_3^{\text{€},z}, g_4^{\text{€},z})$	$\frac{12}{100}$	$\frac{1}{1-\delta} \frac{2}{100}$
	$(g_1^{\text{€},z}, g_2^{\text{€},z}, w_3, g_4^{\text{€},z})$	$\frac{36}{100}$	$\frac{1}{1-\delta} \frac{2}{100}$
	$(g_1^{\text{€},z}, g_2^{\text{€},z}, g_3^{\text{€},z}, w_4)$	$\frac{44}{100}$	$\frac{1}{1-\delta} \frac{2}{100}$
4	$(\mathbf{g_1^{\text{€},z}}, \mathbf{g_2^{\text{€},z}}, \mathbf{g_3^{\text{€},z}}, \mathbf{g_4^{\text{€},z}})$	$\mathbf{0}$	$\mathbf{\frac{1}{1-\delta} \frac{2}{100}}$

Table I: CCs with lump sum refund z in our market example

to implement CCs. For example, we infer from Table I that the clause profile in which only the retailer with the largest capacity adopts a CC with lump sum refund z induces collusive behavior for $\frac{56}{100} \leq \delta < \frac{80}{100}$. Moreover, the collusive outcome can result at $\frac{20}{100} \leq \delta < \frac{56}{100}$ if the two retailers with the largest capacities adopt CCs with lump sum refund z, and at $\frac{8}{100} \leq \delta < \frac{20}{100}$ if the three retailers with the largest capacities adopt such CCs. Finally, for $0 \leq \delta < \frac{8}{100}$, collusion can be realized by all retailers adopting CCs with lump sum refund z.

The key message put forward by Table I is that collusive behavior can be enforced even though only a few of the retailers adopt CCs. For example, if the customers incur no hassle costs and the common discount factor of the retailers belongs to interval $\frac{8}{100} \leq \delta < \frac{80}{100}$, collusion in the market is achieved by partial adoption of MCCs. Remarkably, this observation is not in line with most of the theoretical literature stating that MCCs sustain collusion only if all retailers adopt such clauses and the set of alternative CC options is substantially restricted. As mentioned above, the decisive factor for this divergence is that the time horizon of the competition considered in our setting differs from the one assumed in those studies. Unlike them, we view the competition between the retailers as an infinitely repeated game.

4 Existence, Robustness, and Impossibility

In this section, we point to implications, refinements, and limitations of the results we derived in the previous section. The focus of our previous analysis has been on the clause profiles in which the CC-adopting retailers opt for the CC with the lump sum refund equal to the hassle costs. Let us henceforth denote the set consisting of those clause profiles by $G^{\text{€},z} := \{(g_J^{\text{€},z}, w_{-J}) : J \subseteq I\}$. The first issue we will address in this section is that of existence. We will examine whether collusion can be enforced by such conventional clause profiles regardless of the value of the common discount factor. As was argued above, this holds in the market example of Section 3.4. It remains to prove whether the claim also holds in general.

It turns out that if the hassle and implementation costs are "mild", then there exists at least one perfectly collusive clause profile in $G^{€,z}$ for any common discount factor. Usually, several of those conventional clause profiles prove to be perfectly collusive. Since some of them seem to be more plausible than others, we impose additional requirements on the clause profiles in order to single out the most reasonable ones. As will be seen, these additional requirements lead to a unique solution in $G^{€,z}$. Interestingly, this clause profile has a salient spreading pattern of the CCs.

Finally, we will take a glimpse beyond the clause profiles of $G^{€,z}$ and examine the issue whether other (conventional) clause profiles could induce collusion. By revisiting the market example of SECTION 3.4, we will see that conventional clause profiles with a greater lump sum refund are also perfectly collusive. Remarkably, some of them induce collusion in a more cost-saving way (i.e., with a smaller number of CC-adopting retailers) than the ones of with lump sum refunds equal to the hassle costs. We already know that the latter clause profiles are perfectly collusive regardless of the underlying rationing rule. However, a complete explicit description of all perfectly collusive clause profiles is in general not possible without any further reference to the underlying rationing rule.

■ An Existence Result

The issue of existence is resolved by THEOREMS 6 and 13. These theorems ensure the non-emptiness of solution concept $\mathcal{S}^{\mathrm{m}}$ for sufficiently small implementation and hassle costs.

To see this result, we first note that $f_{\mathrm{crit}}^{g,\delta} \geq \frac{1}{1-\delta}\kappa_1\pi^{\mathrm{m}}$ for any clause profile $g \in G$. Moreover, according to REMARK 9, it holds $\bar{z}_1^{\delta} \geq \bar{z}_1^{0}$ for any $0 \leq \delta < \delta_{\mathrm{crit}}$. Therefore, it follows from THEOREM 13 that collusion can be enforced by clause profiles of $G^{€,z}$ for any common discount factor below δ_{crit} as long as the actual implementation and hassle costs do not exceed $\frac{1}{1-\delta}\kappa_1\pi^{\mathrm{m}}$ and $\bar{z}_1^0$, respectively. Apart from that, we know from THEOREM 6 that collusion can result without any CCs if the common discount factor is greater than or equal to δ_{crit}. Putting these findings together, we obtain the following existence result.

Corollary 14. *If the implementation and hassle costs satisfy $f \leq \kappa_1\pi^m$ and $z \leq \bar{z}_1^0$, respectively, then it holds for any common discount factor δ and any number of retailers n that solution set $\mathcal{S}^m(\delta, f, n, z)$ contains a business policy profile in which all CC-adopting retailers choose the CC with the lump sum refund equal to hassle costs.*

■ A Uniqueness Result

According to the last result, if the hassle and implementation costs are sufficiently small, we find a perfectly collusive clause profile in $G^{€,z}$ for any common discount factor. However, it turns out that $G^{€,z}$ usually contains several perfectly collusive clause profile. For example, consider the market described in SECTION 3.4 at common discount factor $\delta := \frac{1}{2}$. As can be easily inferred from THEOREM 13 and TABLE I, collusion is reached in this situation if the coalition of the CC-adopting retailers belongs to system $\mathscr{J} := \{\{3,4\},\{2,4\},\{1,4\},\{1,2,3\}\}$ and each of the CC-adopting retailers chooses the CC with the lump sum refund equal to the hassle costs.

To select the most plausible ones of these solutions, we impose two additional requirements. These requirements are based on efficiency and robustness considerations and are lexicographically ranked. The primary requirement is cost-efficiency and refers to the total implementation costs of a clause profile, i.e., the sum of the implementation costs of all CC-adopting retailers. A perfectly collusive clause profile is said to be a *cost-efficient* in $G^{€,z}$ if it there is no other perfectly collusive clause profile in $G^{€,z}$ with lower total implementation costs (or synonymously, with a smaller number of CC-adopting retailer). This requirement can be motivated by the argument that (tacitly) concerted competition clause policies might be easier to realize the less retailers are involved.

The secondary requirement is that of robustness. It is required that there be no other cost-efficient clause profile in $G^{€,z}$ with more permissible thresholds regarding the common discount factor or the implementation costs, i.e., with a lower critical discount factor or greater critical implementation costs. A clause profile fulfilling the two criteria is said to be *robustly collusive* in $G^{€,z}$. Applying this refinement, we substantially narrow down the solutions in $G^{€,z}$.

Proposition 15. *Consider $\Gamma(\delta, f, n, z)$ where $\delta < \delta_{crit}$ and $z \leq \bar{z}_1^0$. There exists a $k \in I$ so that clause profile $(g_{I_k}^{€,z}, w_{-I_k})$ is the only robustly collusive one in $G^{€,z}$.*

We obtain from this proposition and COROLLARY 14 a uniqueness result: For any common discount factor, there is a unique robustly collusive clause profile in $G^{€,z}$. Remarkably, this clause profile has a specific spreading pattern of the CCs; it is the one in which the CC-adopting retailers are the retailers with the largest market shares. The robustly collusive clause profiles of $G^{€,z}$ in the market of SECTION 3.4 are printed in bold letters in TABLE I.

■ An Impossibility Result

As shown in COROLLARY 14, we are able to provide a perfectly collusive clause profile in $G^{€,z}$ for any arbitrary common discount factor whenever hassle and implementation costs are sufficiently small. This finding might immediately raise the question whether $G^{€,z}$ encompasses all (conventional and) perfectly collusive clause profiles. To clarify this issue, let us return to the retail market described in SECTION 3.4 and consider several clause profiles in which a coalition of retailers adopts BCCs with lump sum refund $p^{\mathrm{m}} - c = \frac{1}{2} > z$. The critical discount factors and critical implementation costs of these clause profiles are listed in TABLE II.

Clause profile g	Critical discount factor δ_{crit}^g	Critical implementation costs $f_{\text{crit}}^{g,\delta}$
$(w_1, w_2, w_3, b_4^{€,\frac{1}{2}})$	$\frac{56}{100}$	$\frac{1}{1-\delta}\frac{11}{100}$
$(w_1, w_2, b_3^{€,\frac{1}{2}}, w_4)$	$\frac{64}{100}$	$\frac{1}{1-\delta}\frac{9}{100}$
$(w_1, b_2^{€,\frac{1}{2}}, w_3, w_4)$	$\frac{80}{100}$	$\frac{1}{1-\delta}\frac{3}{100}$
$(b_1^{€,\frac{1}{2}} w_2, w_3, w_4)$	$\frac{80}{100}$	$\frac{1}{1-\delta}\frac{2}{100}$
$(w_1, w_2, b_3^{€,\frac{1}{2}}, b_4^{€,\frac{1}{2}})$	0	$\frac{1}{1-\delta}\frac{9}{100}$

Table II: BCCs with lump sum refund $p^{\mathrm{m}} - c$ in our market example

Applying the three criteria stated in REMARK 4, we infer from the values in the first row of TABLE II that the clause profile in which the retailer with the largest capacity adopts a BCC with lump sum refund $p^{\mathrm{m}} - c$ is perfectly collusive whenever the common discount factor satisfies the inequalities $\frac{56}{100} \leq \delta < \frac{80}{100}$. Interestingly, if one compares this result with the result derived in the second row of TABLE I, one comes to the conclusion that it makes no difference whether this retailer adopts a CC with lump sum z or with lump sum $p^{\mathrm{m}} - c$.

As indicated by the second row of TABLE II, collusive behavior can also achieved if the retailer with the second largest capacity adopts such BCC. However, compared to the former clause profile, this holds only for a smaller range of common discount factors and implementation costs.

According to the third and fourth row of TABLE II, the critical discount factor is unchanged at $\delta_{\text{crit}} = \frac{80}{100}$ if a BCC with lump sum refund $p^{\mathrm{m}} - z$ is adopted by one of the two smallest retailers. Therefore, such clause profiles do not cause collusive behavior. This finding is in accord with THEOREM 8 stating that if the total capacity of the CC adopting retailers does not meet the market demand at the monopoly price, then the CCs are collusively ineffective.

The fifth row of TABLE II reveals that for any common discount factor below $\frac{56}{100}$, collusion is sustained by the clause profile in which the two retailers with the largest capacities adopt BCCs with lump sum refund $p^{\mathrm{m}} - c$. The difference between this finding and the findings of TABLE I is striking. According to TABLE I and THEOREM 13, CCs with lump sum refund z have to be implemented by more than two retailers in order to become collusive efficacious for common discount factors lower than $\frac{20}{100}$. Hence, for such discount factors, the total implementation costs (i.e., the sum of the implementation costs) necessary for achieving collusion is less if BCCs with lump sum refund $p^{\mathrm{m}} - c$ are adopted by the two largest retailers.

We conclude from these observations that THEOREM 13 does not provide a complete list of the perfectly collusive clause profile. But even more, there are market situations in which perfectly collusive clause profiles not listed in the theorem are more cost-saving than the ones listed. Therefore, in order to select the most plausible perfectly collusive CCs, it might be desirable to gain an overview of all perfectly collusive CCs.

A characterization of the perfectly collusive clause profiles has already been given in REMARK 4. However, such implicit description of the solution set is of less value as no specific reference regarding the form and spread of the collusive CCs is made. At least, it might be instructive to specify all CCs with lump sum refunds of the solution set. However, as will be set forth next, such specification proves to be an insurmountable exercise without any reference to the rationing rule underlying the residual market demand.

To see this, let us revisit the market example of SECTION 3.4 and calculate the critical discount factor of the clause profile in which both the retailer with the largest and the retailer with the smallest capacity implement a BCC with lump sum refund $p^{\mathrm{m}} - c$. Unlike the critical discount factors of the clause profiles presented in the previous two tables, this threshold depends on the underlying rationing rule. To see this point, TABLE III lists the critical discount factors for the efficient, proportional, and perfect rationing rule.

Clause profile g	Critical discount factor δ^g_{crit}			Critical implementation costs $f^{g,\delta}_{\mathrm{crit}}$
	efficient	proportional	perfect	
$(b_1^{€,\frac{1}{2}}, w_2, w_3, b_4^{€,\frac{1}{2}})$	$\frac{31.25}{100}$	$\frac{45}{100}$	$\frac{56}{100}$	$\frac{1}{1-\delta}\frac{2}{100}$

Table III: Clause profile $(b_1^{€,\frac{1}{2}}, w_2, w_3, b_4^{€,\frac{1}{2}})$ in our market example

To find out whether clause profile $(b_1^{€,\frac{1}{2}}, w_2, w_3, b_4^{€,\frac{1}{2}})$ is perfectly collusive, we apply REMARK 4. Since the implementation costs do not provide an obstacle for the two CC-adopting retailers (i.e., $f \leq \frac{1}{1-\delta}\frac{2}{100}$), it remains to compare the critical discount factors of TABLE III with those of the first and fourth row of TABLE II. In the case that the rationing rule is efficient, the clause profile proves to be perfectly collusive whenever the common discount factor belongs to the bounded right-open interval $[\frac{31.25}{100}, \frac{56}{100}[$. If the rationing rule is proportional, the range of the common discount factor for which the clause profile is perfectly collusive shrinks to the bounded right-open interval $[\frac{45}{100}, \frac{56}{100}[$. Remarkably, this clause profile is never perfectly collusive whenever the perfect rationing rule applies.

We conclude from these findings that it is impossible to specify (all conventional business policy profiles of) the solution set of the market example in SECTION 3.4 without any reference to the underlying rationing rule. This impossibility result is summarized in the following remark.

Remark 16. *It is not always possible to specify all conventional business policy profiles of solution set $\mathcal{S}^m(\delta, f, n, z)$ without any further reference to the rationing rule underlying the residual market demand.*

5 Concluding Remarks

In their empirical study of the incidence and variety of low price guarantees, Arbatskaya et al. (2004, p. 321) point out: "Economic theory is silent on the topic of which guarantee is best because the literature tends to posit static models of equilibrium in which no firm is ever 'surprised'. In this world, both types of guarantees [MCC and BCC] are equally effective at facilitating high prices, and there are no grounds to choose one over the other." The aim of our paper is to remedy this obvious shortcoming in the theoretical literature on low price guarantees. We have modeled the competition in the retail market as an infinite multi-stage game and have examined the form and spread of low price guarantees within this game-theoretical framework.

The starting point of our competition model is a perfectly homogeneous commodity market in which the retailers are identical except for their sales capacities. Their competition clause policies have been considered as binding commitments. They are implemented at the initial stage of the competition game, the so-called clause implementation phase, and remain unchanged in the subsequent stages. In each of those stages, the retailers compete in the commodity market and advertise prices for the commodity. The effective prices the retailers charge for the commodity result from those advertised prices and the implemented competition clause policies. The stages after the clause implementation phase constitute the so-called price competition phase.

The implementation and exertion of competition clauses (CCs) might be costly in real life. Therefore, such costs have been taken into account in our framework. More specifically, we have assumed that the retailers adopting CCs incur the same fixed implementation costs and the customers exercising CCs incur the same hassle costs per purchased unit of the commodity. Since a substantial part of the theoretical literature does not consider hassle costs, we have also examined the case of no hassle costs for the sake of comparability.

Our multi-stage competition game is one with perfect information. When the retailers decide about the prices at some stage in the price competition phase, they know all previous choices of their competitors. The consumers are fully informed about the advertised prices and adopted competition clause policies before they go shopping. The assumption of perfect information has serious consequences. It entails that CCs are not implemented for other reasons than facilitating collusion.

The end date of the price competition phase might, however, be unknown to the retailers. For this reason, we have viewed the price competition between them as an infinitely repeated game. Future profits are discounted. In order to also include the cases in which the end date of the price competition phase is foreseeable for the retailers, we have allowed for the possibility that the common discount factor can take the value of zero.

To make predictions of the form and spread of collusive CCs, we have applied the solution concept of subgame perfectness. Additionally, to simplify our analysis, we have assumed that the grim-trigger price policies are pursued by the retailers in the price competition phase whenever these policies constitute a subgame perfect equilibrium in this subgame. Otherwise, the retailers pursue the competitive price policies, i.e., they advertise prices equal to the marginal costs in each stage of the price competition phase regardless of which prices have been advertised in the previous stages.

The first important result of our analysis has been that the Perfect Folk Theorem by Friedman (1971) can be transferred to our extended Bertrand competition game: Whenever the common discount is sufficiently close to 1, collusive behavior of the retailers is enforceable without adopting CCs (cf. Theorem 6). In such cases, the threats inherent in the grim-trigger price policies suffice to induce collusion. However, those threats prove to be too weak whenever the common discount factor is not sufficiently close to 1. One of the primary objectives of our paper is to examine whether collusive behavior can be induced by the adoption of CCs for such common discount factors.

Our analysis shows that CCs can be used by the retailers as a device facilitating collusion for any of those discount factors. However, the collusive efficacy of CCs depends on several conditions regarding the costs, spread, and form of the CCs. Regarding the latter, we have put special emphasis on BCCs with lump sum refunds. Such BCCs are promises of the retailers that their customers are entitled to purchase the commodity at the lowest advertised price in the market minus a refund in the amount of x € whenever the retailers fail to advertise the lowest price in the market.

One necessary condition for the collusive efficacy of CCs is that the implementation and hassle costs of the CCs are sufficiently small (cf. THEOREM 10). If the implementation costs exceed the total collusive profit of the largest retailer, then it proves to be unprofitable for any retailer to adopt CCs even though such policy would induce collusion. On the other hand, if the implementation costs do not exceed the total collusive profit of the smallest retailer, then the implementation costs are no longer an obstacle for using CCs as a facilitating device.

High hassle costs render any CC collusively ineffective. This follows from our requirement that the sales price guaranteed by a CC do not fall below the marginal costs whenever the advertised price is not below the marginal costs. This assumption has been imposed in order to avoid unsustainable situations in which the CC-adopting retailers incur losses if their customers exercise the CCs. However, if the hassle costs are "mild" so that any worthwhile price reduction by the smallest retailer can be immediately undercut by the BCCs of its competitors, collusion becomes enforceable.

To sum up, we agree with Hviid and Shaffer (1999) that the MCC becomes collusively ineffective in homogeneous retail markets whenever hassle costs exit (cf. THEOREM 11). Nevertheless, we cannot confirm their claim (see Section 4.4 in Hviid and Shaffer, 1999) that other CCs are also collusively ineffective in this case. Indeed, it has been demonstrated that the collusive outcome can be enforced if a sufficiently large coalition of retailers offers BCCs with lump sum refunds equal to the hassle costs (cf. THEOREM 13). The reason for this divergence is due to different assumptions regarding the timing of the competition. Unlike our setting, Hviid and Shaffer (1999) view the retail market as a one-stage game in which the retailers choose the competition clause policies and the prices simultaneously.

A further necessary condition for the collusive efficacy of CCs is that the total capacity of the CC-adopting retailers has to be large enough to meet the market demand at the collusive price (cf. THEOREM 8). However, it is in general not required that all retailers have to adopt CCs in order to enforce collusion, as claimed in most of the earlier theoretical papers on the CCs like in Doyle (1988) and Logan and Lutter (1989). Some research papers agree with ours that partial adoption of CCs is consistent with collusive intentions. However, those papers take for granted some asymmetries in the market such as sequential price-setting or heterogeneous cost or demand conditions. A merit of our paper is that one has not to resort to such asymmetries to reconcile the theory of collusive CCs with the striking empirical finding that in most commodity markets only a few retailers adopt CCs.

From the above findings, we have inferred a general collusion existence theorem. If the hassle and implementation costs are sufficiently small, then collusion is enforceable by a coalition of retailers adopting the BCC with the lump sum refund equal to the hassle costs. This holds regardless of the value of the common discount factor (cf. COROLLARY 14). However, usually there are several of such collusion inducing coalitions where some exhibit a higher degree of plausibility than others.

To select the most plausible ones, we have substantially refined the solution concept of subgame perfectness. Our primary selection criterion is cost-efficiency. It requires that the collusive outcome be induced by minimal total implementation costs. This efficiency criterion could be justified by the argument that tacit collusion might be more easily enforced the less the number of the CC-adopting retailers is. In the second place, we require that the coalition induce the lowest critical discount factor and the highest critical implementation costs. That means, it has to be the most robust with respect to decreases in the common discount factor and increases in the implementation costs.

Applying these refinement criteria, we have obtained a unique spreading pattern of collusively effective CCs. It has turned out that if all CC-adopting retailers choose the BCC with the lump sum refund equal to the hassle costs, then the most robust coalition is the one in which the retailers with the largest market shares offer the CCs (cf. PROPOSITION 15). Remarkably, this spreading pattern proves to be independent of the rationing rule underlying the residual market demand.

The existence of collusive outcomes has been established by the (partial) adoption of BCCs with lump sum refunds equal to the hassle costs. Nevertheless, it might be desirable to provide an explicit description of all collusively effective forms of CCs. However, this task has proved to be insurmountable without further assumptions regarding the rationing rule (cf. REMARK 16). This impossibility result is due to the ambiguity that some forms of CCs are collusively efficacious for some rationing rules, but not for others.

Although our theoretical analysis of the collusive efficacy of CCs is more general than most previous ones, it has nevertheless some significant limitations which could be addressed in future research projects. First, our analysis has been focused on the collusive efficacy of CCs with lump sum refunds. However, as put forward by Arbatskaya et al. (2004), two other forms of CCs are also (or even more) common in real commercial life, the CCs with a fixed refund factor on price differences ("We promise that if we do not offer the lowest price in the market, we'll beat it by an amount equal to x % of the difference between our announced price and the lowest price in the market.") and with a refund factor on the minimum price ("We promise that if we do not offer the lowest price in the market, we'll beat it by x % ."). It might be interesting to know whether (the partial adoption of) such CCs also induce collusion in our setting.

However, as we know from REMARK 16, a complete description of all collusively efficacious CCs is in general impossible without any further reference on the underlying rationing rule. A future research project could address this problem. By tightening the solution concept and considering specific settings of our competition model, it might be possible to explicitly specify at least all common forms of CCs which induce collusion. For example, a promising approach might be to narrow down the solution set to the most robust solutions; to impose additional, but reasonable requirements on the rationing rules; and to focus on particular market structures.

Another worthwhile research issue might be to examine whether the most robust collusive solutions have a common spreading pattern of the CCs. We already derived in PROPOSITION 15 that if all CC-adopting retailers choose BCCs with the lump sum refund equal to the hassle costs, the spreading pattern of the CCs in the most robust solution is the one in which the retailers with the largest market shares adopt the CCs. It is open question whether this salient spreading pattern of the collusive CCs holds in general.

If this claim can be confirmed, one might have a simple demarcation criterion indicating whether CCs are used by the retailers as a device facilitating collusion or for some other reasons. Usually, the partial adoption of competition clauses has been interpreted as an indication that the adoption of CCs relies on intentions others than the collusive one, e.g., to signal low price or to price differentiate between uninformed loyals and well-informed shoppers; regarding this argument, see e.g. the expositions in SECTION 3 in Moorthy and Winter (2006). However, as set forth in our paper, this conclusion might not be justified in general. Rather, if the above claim holds, the market shares of the CC-adopting retailers should be used as an indicative criterion for collusion. For example, collusive intentions might prevail in markets where the CCs are adopted by dominant retailers. If only fringe retailers offer CCs, the CCs might be implemented for other motives than collusion.

Appendix: Proofs

Proof of Remark 1.

(a) To prove this claim, we consider the situation in which clause profile g has been adopted and retailer i advertises price $q_i \neq p^{\mathrm{m}}$ while its competitors advertise prices p^{m}_{-i}.

- Suppose $q_i > c$. Due to Assumption (G), the effective sales price at retailer i amounts to $q_i^{\mathrm{s}} \geq c$ so that $\pi_i^g(q_i, p^{\mathrm{m}}_{-i}) = (q_i^{\mathrm{s}} - c)X_i(q^{\mathrm{p}}) \geq 0$. We conclude from this weak inequality that

$$0 \leq \sup_{c < q_i \neq p^{\mathrm{m}}} \pi_i^g(q_i, p^{\mathrm{m}}_{-i}) \leq \sup_{q_i \neq p^{\mathrm{m}}} \pi_i^g(q_i, p^{\mathrm{m}}_{-i}).$$

- Suppose $q_i \leq c$. Due to Assumption (G), the effective sales price at retailer i is $q_i^{\mathrm{s}} = q_i$ so that $\pi_i^g(q_i, p^{\mathrm{m}}_{-i}) = (q_i - c)X_i(q^{\mathrm{p}}) \leq 0$. Note that if $q_i = c$, we have $\pi_i^g(q_i, p^{\mathrm{m}}_{-i}) = (q_i - c)X_i(q^{\mathrm{p}}) = 0$. We conclude that

$$\sup_{q_i \leq c} \pi_i^g(q_i, p^{\mathrm{m}}_{-i}) = 0.$$

By bringing the above two results together, we obtain

$$0 \leq \sup_{c < q_i \neq p^{\mathrm{m}}} \pi_i^g(q_i, p^{\mathrm{m}}_{-i}) = \sup_{q_i \neq p^{\mathrm{m}}} \pi_i^g(q_i, p^{\mathrm{m}}_{-i}).$$

(b) To prove this claim, we consider the situation in which clause profile g has been adopted and retailer i advertises price $q_i \neq p^{\mathrm{m}}$ while its competitors advertise prices p^{m}_{-i}.

- Suppose $q_i > p^{\mathrm{m}}$. Since retailer i's competitors are assumed to advertise collusive price p^{m}, Assumption (G) implies that the effective purchase price charged by them is equal to p^{m}. The effective purchase price at retailer i can be either above or equal to or below p^{m}.

  In the first case, Assumptions (D2), (D4), (R1), and (I) imply that the demand for the product offered by retailer i is zero. Hence, retailer i earns a profit $\pi_i^g(q_i, p^{\mathrm{m}}_{-i}) = 0$. We conclude that

$$\pi_i^g(q_i, p^{\mathrm{m}}_{-i}) < (p^{\mathrm{m}} - c)\min\{k_i, D(p^{\mathrm{m}})\}$$

  if $q_i^{\mathrm{p}} > p^{\mathrm{m}}$.

  In the second case, Assumptions (R1) and (I) imply that retailer i sells quantity $X_i(q^{\mathrm{p}}) = \kappa_i D(p^{\mathrm{m}})$ and, thus, earns a profit $\pi_i^g(q_i, p^{\mathrm{m}}_{-i}) = (p^{\mathrm{m}} - z - c)\kappa_i D(p^{\mathrm{m}})$. Since $z \geq 0$, we have $\pi_i^g(q_i, p^{\mathrm{m}}_{-i}) \leq (p^{\mathrm{m}} - c)\kappa_i D(p^{\mathrm{m}})$. Moreover, Assumption (D4) entails $D(p^{\mathrm{m}}) < K$ so that $(p^{\mathrm{m}} - c)\kappa_i D(p^{\mathrm{m}}) < (p^{\mathrm{m}} - c)\max\{k_i, D(p^{\mathrm{m}})\}$. We conclude that

$$\pi_i^g(q_i, p^{\mathrm{m}}_{-i}) < (p^{\mathrm{m}} - c)\max\{k_i, D(p^{\mathrm{m}})\}$$

  if $q_i^{\mathrm{p}} = p^{\mathrm{m}}$.

  Finally, assume the last case. Due to Assumptions (R1) and (I), retailer i sells quantity $X_i(q^{\mathrm{p}}) = \min\{k_i, D(q_i^{\mathrm{p}})\}$ so that its profit amounts to $\pi_i^g(q_i, p^{\mathrm{m}}_{-i}) = (q_i^{\mathrm{p}} - z - c)\min\{k_i, D(q_i^{\mathrm{p}})\}$. Since $z \geq 0$, it holds $\pi_i^g(q_i, p^{\mathrm{m}}_{-i}) \leq (q_i^{\mathrm{p}} - c)\min\{k_i, D(q_i^{\mathrm{p}})\}$. Moreover, $q_i^{\mathrm{p}} < p^{\mathrm{m}}$ entails $(q_i^{\mathrm{p}} - c)\min\{k_i, D(q_i^{\mathrm{p}})\} < (p^{\mathrm{m}} - c)\min\{k_i, D(p^{\mathrm{m}})\}$ so that

$$\pi_i^g(q_i, p^{\mathrm{m}}_{-i}) < (p^{\mathrm{m}} - c)\min\{k_i, D(p^{\mathrm{m}})\}.$$

- Suppose $q_i < p^{\mathrm{m}}$. In this case, Assumption (G) implies that the effective purchase price at retailer i corresponds to the advertised one, i.e., $q_i^{\mathrm{p}} = q_i$. Since retailer i's competitors advertise collusive price p^{m}, it also follows from Assumption (G) that the effective purchase prices charged by them are somewhere between c and p^{m}. Due to Assumptions (R1) and (I), the sales of retailer i is bounded from above by $X_i(q^{\mathrm{p}}) \leq \min\{k_i, D(q_i)\}$. Since $q_i < p^{\mathrm{m}}$, we obtain

$$\pi_i^g(q_i, p^{\mathrm{m}}_{-i}) \leq (q_i - c)\min\{k_i, D(q_i)\} \leq (p^{\mathrm{m}} - c)\min\{k_i, D(p^{\mathrm{m}})\}.$$

Summing up the above results, we have established

$$\sup_{q_i \neq p^{\mathrm{m}}} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) \leq (p^{\mathrm{m}} - c) \min\{k_i, D(p^{\mathrm{m}})\}. \qquad \square$$

Proof of Remark 2. Let us suppose clause profile w has been implemented, i.e., none of the retailers has adopted a CC. Moreover, consider some retailer i advertising price $q_i < p^{\mathrm{m}}$. Since its competitors are assumed to advertise collusive price p^{m}, retailer i underbids all of them. Due to ASSUMPTIONS (R1) and (I), retailer i's sales are then given by $X_i(q_i, p_{-i}^{\mathrm{m}}) = \min\{k_i, D(q_i)\}$ so that it earns a profit $\pi_i(q_i, p_{-i}^{\mathrm{m}}) = (q_i - c) \min\{k_i, D(q_i)\}$.

- Suppose $k_i \leq D(p^{\mathrm{m}})$. It follows from ASSUMPTION (D2) that $\pi_i(q_i, p_{-i}^{\mathrm{m}}) = (q_i - c)k_i$. This in turn implies $\sup_{q_i < p^{\mathrm{m}}} \pi_i(q_i, p_{-i}^{\mathrm{m}}) = (p^{\mathrm{m}} - c)k_i$. Applying REMARK 1(b), we even obtain $\sup_{q_i \neq p^{\mathrm{m}}} \pi_i(q_i, p_{-i}^{\mathrm{m}}) = (p^{\mathrm{m}} - c)k_i$. Since

$$\frac{1}{1-\delta} \kappa_i \, \pi(p^{\mathrm{m}}) \;\geq\; \sup_{q_i \neq p^{\mathrm{m}}} \pi_i(q_i, p_{-i}^{\mathrm{m}}) \quad \Longleftrightarrow \quad \delta \;\leq\; 1 - \frac{D(p^{\mathrm{m}})}{K},$$

  it holds $\delta_{i,\mathrm{crit}} = \min \Delta_i = 1 - \frac{D(p^{\mathrm{m}})}{K}$.

- Suppose $k_i > D(p^{\mathrm{m}})$. It follows from ASSUMPTION (D1) that there is some $\epsilon > 0$ so that $k_i > D(q_i)$ for any $p^{\mathrm{m}} - \epsilon < q_i < p^{\mathrm{m}}$. This in turn implies $X_i(q_i, p_{-i}^{\mathrm{m}}) = D(q_i)$ for any $p^{\mathrm{m}} - \epsilon < q_i < p^{\mathrm{m}}$. Moreover, ASSUMPTION (D1) ensures $\lim_{q_i \uparrow p^{\mathrm{m}}} X_i(q_i, p_{-i}^{\mathrm{m}}) = D(p^{\mathrm{m}})$ and, thus, $\lim_{q_i \uparrow p^{\mathrm{m}}} \pi_i(q_i, p_{-i}^{\mathrm{m}}) = \pi^{\mathrm{m}}$. We conclude from this result that $\sup_{q_i < p^{\mathrm{m}}} \pi_i(q_i, p_{-i}^{\mathrm{m}}) = \pi^{\mathrm{m}}$. Applying REMARK 1(b), we even have $\sup_{q_i \neq p^{\mathrm{m}}} \pi_i(q_i, p_{-i}^{\mathrm{m}}) = \pi^{\mathrm{m}}$. Since

$$\frac{1}{1-\delta} \kappa_i \, \pi^{\mathrm{m}} \;\geq\; \sup_{q_i \neq p^{\mathrm{m}}} \pi_i(q_i, p_{-i}^{\mathrm{m}}) \quad \Longleftrightarrow \quad \delta \;\leq\; 1 - \kappa_i,$$

  it holds $\delta_{i,\mathrm{crit}} = \min \Delta_i = 1 - \kappa_i$.

Summing up the above two cases, we have shown that $\sup_{q_i \neq p^{\mathrm{m}}} \pi_i(q_i, p_{-i}^{\mathrm{m}}) = (p^{\mathrm{m}} - c) \min\{k_i, D(p^{\mathrm{m}})\}$ and, thus, $\delta_{i,\mathrm{crit}} = 1 - \max\{\kappa_i, \frac{D(p^{\mathrm{m}})}{K}\}$.

Consider now some arbitrary clause profile g. Due to REMARK 1(b) and the above result, it holds $\sup_{q_i \neq p^{\mathrm{m}}} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) \leq (p^{\mathrm{m}} - c) \min\{k_i, D(p^{\mathrm{m}})\} = \sup_{q_i \neq p^{\mathrm{m}}} \pi_i(q_i, p_{-i}^{\mathrm{m}})$. Therefore, we have

$$\begin{aligned} \Delta_i \quad &:= \quad \left\{ \delta \in [0,1[: \frac{1}{1-\delta} \kappa_i \, \pi(p^{\mathrm{m}}) \;\geq\; \sup_{q_i \neq p^{\mathrm{m}}} \pi_i(q_i, p_{-i}^{\mathrm{m}}) \right\} \\ &\subseteq \quad \left\{ \delta \in [0,1[: \frac{1}{1-\delta} \kappa_i \, \pi(p^{\mathrm{m}}) \;\geq\; \sup_{q_i \neq p^{\mathrm{m}}} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) \right\} =: \Delta_i^g. \end{aligned}$$

It immediately follows from $\Delta_i \subseteq \Delta_i^g$ that $\delta_{i,\mathrm{crit}} \geq \delta_{i,\mathrm{crit}}^g$. $\square$

Proof of Remark 3. To prove this remark, we resort to THEOREM 3.2. of Fudenberg and Tirole (1991). This theorem states that if a multi-stage game is continuous at infinity, then a strategy profile is subgame perfect if, and only if, none of the players can increase its payoff in any subgame through a one-shot deviation. This characterization of subgame perfectness is known as the ONE-SHOT DEVIATION PRINCIPLE.

Our first step is to establish that this theorem is indeed applicable to our competition game. Afterwards, the ONE-SHOT DEVIATION PRINCIPLE is formalized. We then apply this principle to show that (i) the grim-trigger price policy profile t^g is subgame perfect in Γ^g if, and only if, $\delta \geq \delta_{\mathrm{crit}}^g$, and (ii) the competitive price policy profile c^g is subgame perfect in subgame Γ^g for any $\delta \geq 0$. With these results at hand, the proof of the "if"- and "only if"-part of the remark follows immediately.

Our competition game Γ is *continuous at infinity* if for any $\epsilon > 0$, any retailer $i \in I$, and any outcome $o \in O$, there exists a period t_0 so that

$$|\Pi_i(\tilde{o}) - \Pi_i(o)| < \epsilon$$

for any outcome $\tilde{o} \in O$ satisfying $\tilde{o}_t = o_t$ for any period $t \leq t_0$. To establish this property, let us consider some $\epsilon > 0$, some retailer $i \in I$, and some outcome $o := (g, q^0, q^1, \dots) \in O$. We note that, due to ASSUMPTIONS (D1),(D2), (R1) and (I), the one-stage profit π_i is bounded. More precisely, it holds $-cD(0) \leq \pi_i(q) \leq \pi^{\mathrm{m}}$ for any profile $q \in \mathbb{R}_+^I$ of advertised prices. Let us define $\eta := \pi^{\mathrm{m}} + cD(0)$. Obviously, it holds $|\pi_i^g(\tilde{q}) - \pi_i^g(q)| \leq \eta$ for any $q, \tilde{q} \in \mathbb{R}_+^I$ and $g \in G$. Choose some sufficiently large period $t_0 \geq 0$ so that $\frac{\delta^{t_0}}{1-\delta}\eta < \epsilon$. Moreover, consider some outcome $\tilde{o} := (\tilde{g}, \tilde{q}^0, \tilde{q}^1, \dots) \in O$ which is consistent with outcome o up to period $t_0 - 1$, i.e, $\tilde{g} = g$ and $\tilde{q}^t = q^t$ for any period $0 \leq t \leq t_0 - 1$. Since

$$\begin{aligned}
|\Pi_i(\tilde{o}) - \Pi_i(o)| &= \left| \sum_{t=0}^{\infty} \delta^t \pi_i^g(\tilde{q}^t) - \sum_{t=0}^{\infty} \delta^t \pi_i^g(q^t) \right| \\
&= \left| \sum_{t=t_0}^{\infty} \delta^t \left(\pi_i^g(\tilde{q}^t) - \pi_i^g(q^t) \right) \right| \\
&\leq \sum_{t=t_0}^{\infty} \delta^t \left| \pi_i^g(\tilde{q}^t) - \pi_i^g(q^t) \right| \\
&\leq \frac{\delta^{t_0}}{1-\delta}\eta \\
&< \epsilon \,,
\end{aligned}$$

we have shown that competition game Γ satisfies continuity at infinity.

A business policy $\tilde{s}_i$ in competition game Γ is said to be a *one-shot deviation* from business policy s_i if $\tilde{s}_i$ deviates from s_i at a single history, i.e., there is some history h so that $\tilde{s}|_{H\setminus\{h\}} = s|_{H\setminus\{h\}}$ and $\tilde{s}(h) \neq s(h)$. A strategy profile $s := (s_i)_{i\in I}$ is said to satisfy the *one-shot deviation property* if none of the players gains by a one-shot deviation in any subgame of Γ, or more formally, if

$$\Pi_i(o^h(s)) \geq \Pi_i(o^h(\tilde{s}))$$

for any strategy profile $\tilde{s} := (\tilde{s}_i, s_{-i}) \in S$ where $\tilde{s}_i$ is a one-shot deviation from s_i, any player $i \in I$, and any history $h \in H$. We remark that this property can be described in a simpler way. As can be easily shown, it suffices to demonstrate that the above weak inequality is satisfied for any strategy profile $\tilde{s} := (\tilde{s}_i, s_{-i}) \in S$ where $\tilde{s}_i$ is a one-shot deviation from s_i at history h.

According to THEOREM 3.2 of Fudenberg and Tirole (1991), subgame perfectness can be checked by the one-shot deviation property. Note, since our competition game Γ is continuous at infinity, any of its subgames is continuous at infinity, too. Therefore, it is admissible to apply the ONE-STAGE DEVIATION PRINCIPLE to any subgame of Γ. In the following, we do this for the subgames Γ^g.

We conclude from these considerations that price policy profile s^g proves to be subgame perfect in Γ^g if, and only if, none of the retailers can increase its payoff by an one-shot deviation from s^g in some subgame of Γ^g. This in turn means that s^g is subgame perfect in Γ^g if, and only if, it holds for any retailer $i \in I$ that $\Pi_i(o^h(s)) \geq \Pi_i(o^h(\tilde{s}))$ is satisfied for some business policy profile s inducing s^g in Γ^g and any business policy profile $\tilde{s} := (\tilde{s}_i, s_{-i})$ which is a one-shot deviation of s_i at some non-initial history $h \in H_g^{t_0}$ (i.e., h is a history up to period $t_0 \geq -1$ according to which clause profile g has been adopted in the implementation phase).

Let us first prove claim (i). For this purpose, we pick some clause profile $g \in G$ and consider some business policy profile s inducing the grim-trigger price policy profile $t^g := (t_j^g)_{j\in I}$ in subgame Γ^g. Moreover, we choose some period $t_0 \geq -1$ and some history $h \in H_g^{t_0}$.

- Suppose either $t_0 = -1$ or $h_{j,t}^{t_0} = p^{\mathrm{m}}$ for any $j \in I$ and any $0 \leq t \leq t_0$. The outcome induced by history h and business policy profile s is $o^h(s) = (h, p^{\mathrm{m}}, p^{\mathrm{m}}, p^{\mathrm{m}}, \dots)$. Pick some retailer $i \in I$ and suppose now that retailer i pursues business policy $\tilde{s}_i$ which is one-shot deviation from business policy s_i at non-initial history $h \in H_g^{t_0}$. Let $\tilde{s}_i^g$ be the price policy in subgame Γ^g induced by business policy $\tilde{s}_i$. Obviously, $\tilde{s}^g$ differs from grim-trigger price policy t_i^g only at history h. We define $q_i^{t_0+1} := \tilde{s}_i^g(h)$. By construction, $q_i^{t_0+1} \neq t_i^g(h) = p^{\mathrm{m}}$. The outcome induced by history h and business policy profile $\tilde{s} := (\tilde{s}_i, s_{-i})$ is $o^h(\tilde{s}) = (h, (q_i^{t_0+1}, p_{-i}^{\mathrm{m}}), c, c, \dots)$.

  Hence, it holds

$$
\begin{aligned}
& & \Pi_i(o^h(s)) &\geq \Pi_i(o^h(\tilde{s})) \\
&\iff & \sum_{t=0}^{\infty} \delta^t \pi_i^g(p^{\mathrm{m}}) &\geq \left(\sum_{t=0}^{t_0} \delta^t \pi_i^g(p^{\mathrm{m}}) \right) + \delta^{t_0+1} \pi_i^g(q_i^{t_0+1}, p_{-i}^{\mathrm{m}}) \\
&\iff & \frac{\delta^{t_0+1}}{1-\delta} \pi_i^g(p^{\mathrm{m}}) &\geq \delta^{t_0+1} \pi_i^g(q_i^{t_0+1}, p_{-i}^{\mathrm{m}}) \\
&\iff & \frac{1}{1-\delta} \kappa_i \, \pi^{\mathrm{m}} &\geq \pi_i^g(q_i^{t_0+1}, p_{-i}^{\mathrm{m}}).
\end{aligned}
$$

  Since $q_i^{t_0+1} \neq p^{\mathrm{m}}$ can be arbitrarily chosen, a one-shot deviation from the grim-trigger price policy at history h proves to be not profitable for retailer i in subgame Γ^h if, and only if, the common discount factor δ satisfies

$$
\frac{1}{1-\delta} \kappa_i \, \pi^{\mathrm{m}} \geq \sup_{q_i \neq p^{\mathrm{m}}} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}).
$$

- Suppose $t_0 \geq 0$ and $h_{j,t}^{t_0} \neq p^{\mathrm{m}}$ for some $j \in I$ and some $0 \leq t \leq t_0$. The outcome induced by history h and business policy profile s is $o^h(s) := (h, c, c, c, \dots)$. Pick some retailer $i \in I$ and suppose now that retailer i pursues business policy $\tilde{s}_i$ which is one-shot deviation from business policy s_i at non-initial history $h \in H_g^{t_0}$. Let $\tilde{s}_i^g$ be the price policy in subgame Γ^g induced by business policy $\tilde{s}_i$. Obviously, $\tilde{s}_i^g$ differs from grim-trigger price policy t_i^g only at history h. As above, we define $q_i^{t_0+1} := \tilde{s}_i^g(h)$. By construction, $q_i^{t_0+1} \neq t_i^g(h) = c$. The outcome induced by history h and business policy profile $\tilde{s} := (\tilde{s}_i, s_{-i})$ is $o^h(\tilde{s}) = (h, (q_i^{t_0+1}, c_{-i}), c, c, \dots)$.

  Note that Assumptions (D2), (D4), (R1), and (I) entail $0 \geq \pi_i^g(q_i^{t_0+1}, c_{-i})$. Therefore, we obtain

$$
\begin{aligned}
& \Pi_i(o^h(s)) - \Pi_i(o^h(\tilde{s})) \\
= \; & \sum_{t=0}^{t_0} \delta^t \pi_i^g(h_t) - \left(\left(\sum_{t=0}^{t_0} \delta^t \pi_i^g(h_t) \right) + \delta^{t_0+1} \pi_i^g(q_i^{t_0+1}, c_{-i}) \right) \\
= \; & - \delta^{t_0+1} \pi_i^g(q_i^{t_0+1}, p_{-i}^{\mathrm{m}}) \\
\geq \; & 0.
\end{aligned}
$$

  Since $q_i^{t_0+1} \neq p^{\mathrm{m}}$ can be arbitrarily chosen, a one-shot deviation from the grim-trigger price policy at history h is never profitable for retailer i in subgame Γ^h regardless of the common discount factor $\delta \geq 0$.

Summing up the above two cases, one-shot deviations from the grim-trigger price policy are not profitable for retailer i in any subgame of Γ^g if, and only if, common discount factor δ satisfies the weak inequality

$$
\frac{1}{1-\delta} \kappa_i \, \pi^{\mathrm{m}} \geq \sup_{q_i \neq p^{\mathrm{m}}} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}),
$$

or equivalently, $\delta \geq \delta_{i,\mathrm{crit}}^g$. From this weak inequality, we conclude that none of the retailers has an incentive in some subgame of Γ^g to choose a one-stage deviation from the grim-trigger price policy if, and only if, $\delta \geq \delta^g_{\text{crit}} := \max\{\delta^g_{i,\text{crit}} : i \in I\}$. Resorting to the One-Shot Deviation Principle, we conclude that the grim-trigger price policy profile is subgame-perfect in subgame Γ^g as long as $\delta \geq \delta^g_{\text{crit}}$.

Let us now prove claim (ii). For this purpose, pick some clause profile $g \in G$ and some business policy profile s inducing the competitive price policy $c^g := (c^g_j)_{j\in I}$ in subgame Γ^g. Moreover, choose some period $t_0 \geq -1$ and some history $h \in H^{t_0}_g$. Obviously, the outcome induced by history h and business policy profile s is $o^h(s) = (h, c, c, c, \dots)$.

Choose some retailer i and suppose now that retailer i pursues business policy $\tilde{s}_i$ which is one-shot deviation from business policy s_i at non-initial history $h \in H^{t_0}_g$. Let $\tilde{s}^g_i$ be the price policy in subgame Γ^g induced by business policy $\tilde{s}_i$. Obviously, $\tilde{s}^g_i$ differs from competitive price policy c^g_i only at history h. We define $q^{t_0+1}_i := \tilde{s}_i(h)$. By construction $q^{t_0+1}_i \neq c^g_i(h) = c$. The outcome induced by h and business policy profile $\tilde{s} := (\tilde{s}_i, s_{-i})$ is $o^h(s) = (h, (q^{t_0}_i, c_{-i}), c, c, \dots)$.

Due to Assumptions (D2), (D4), (R1), and (I), it holds $0 \geq \pi^g_i(q^{t_0+1}_i, c_{-i})$. Hence, we obtain

$$\begin{aligned}
& \Pi_i(o^h(s)) \ - \ \Pi_i(o^h(\tilde{s})) \\
= \ & \sum_{t=0}^{t_0} \delta^t \pi^g_i(h_t) \ - \ \left(\left(\sum_{t=0}^{t_0} \delta^t \pi^g_i(h_t) \right) + \delta^{t_0+1} \pi^g_i(q^{t_0+1}_i, c_{-i}) \right) \\
= \ & - \delta^{t_0+1} \pi^g_i(q^{t_0+1}_i, p^{\text{m}}_{-i}) \\
\geq \ & 0.
\end{aligned}$$

Since $q^{t_0+1}_i \neq p^{\text{m}}$ can be arbitrarily chosen, a one-shot deviation from the competitive price policy at history h is never profitable for retailer i in subgame Γ^h regardless of common discount factor $\delta \geq 0$. This holds for any retailer i so that none of them has an incentive to choose a one-stage deviation from the competitive price policy in any subgame of Γ^g regardless of the value of common discount factor δ. Applying the One-Shot Deviation Principle, we conclude that the competitive price policy is subgame-perfect in subgame Γ^g for any common discount factor $\delta \geq 0$.

Summing up, we have shown that (i) the grim-trigger price policy profile in subgame Γ^g is subgame perfect if, and only if, $\delta \geq \delta^g_{\text{crit}}$, and (ii) the competitive price policy profile in subgame Γ^g is subgame perfect for any $\delta \geq 0$. Resorting to these results, we will prove the "if"- and "only if"-part of this remark.

("*if*") Let $\hat{s} := (\hat{s}_i)_{i\in I}$ be a strategy profile satisfying Properties (T1) and (T2). Applying our result (i) to Property (T1), we obtain

$$\hat{s}^g = \begin{cases} t^g & \text{if } t^g \text{ is a subgame perfect equilibrium in } \Gamma^g, \\ c^g & \text{otherwise} \end{cases}$$

for any clause profile $g \in G$. We also know from our result (ii) that competitive price policy profile c^g is subgame perfect in any subgame Γ^g. Therefore, business policy profile $\hat{s}$ induces a subgame perfect strategy profile in any subgame Γ^g. This in turn implies that any one-stage deviation from $\hat{s}$ at histories $h \in H^{t_0}$ where $t_0 \geq -1$ is not profitable in subgame Γ^h. According to the One-Shot Deviation Principle, in order to establish that $\hat{s}$ is subgame perfect in Γ, it remains to show that one-stage deviations from $\hat{s}_i$ at initial history $h := \emptyset$ are also not profitable for any retailer i. However, this is guaranteed by Property (T2).

("*only if*") Let $\hat{s} := (\hat{s}_i)_{i\in I} \in \mathcal{S}^{\text{g}}(\Gamma)$. Applying our result (i) to the construction of $\hat{s}$, we conclude that

$$\hat{s}^g = \begin{cases} t^g & \text{if } t^g \text{ if } \delta \geq \delta^g_{\text{crit}}, \\ c^g & \text{otherwise} \end{cases}$$

and, thus, Property (T1) is satisfied. Moreover, the One-Shot Deviation Principle immediately implies Property (T2). □

Proof of Remark 4. ("*if*") Consider the business policy profile $\hat{s} := (\hat{g}, (\hat{s}^g)_{g\in G})$ satisfying Property (T1). By Property (M1), the grim-trigger price policies are applied by all retailers in subgame $\Gamma^{\hat{g}}$. This entails that business policy profile $\hat{s}$ induces the collusive outcome and, thus, any retailer i earns a total profit

$$\Pi_i(o(\hat{s})) = \frac{1}{1-\delta}\,\kappa_i\,\pi(p^{\mathrm{m}}) - \mathbf{1}_{C_i}(g_i)\,f.$$

at this business policy profile.

Let us now consider a business policy profile $\tilde{s} := (\tilde{s}_i, \hat{s}_{-i}))$ where $\tilde{s}_i := (\tilde{g}_i, (\hat{s}_i^g)_{g\in G})$ and $\tilde{g}_i \neq g_i$. In words, the only difference between business policy profiles $\hat{s}$ and $\tilde{s}$ is that retailer i pursues a different competition clause policy. In the following, we compare the total profits retailer i earns at those business policy profiles:

- If $\delta \geq \delta_{\mathrm{crit}}^{\tilde{g}}$ as well as $i \in C(\hat{g})$ and $i \in C(\tilde{g})$, then both business policy profiles induce the collusive outcome. Since retailer i adopts a CC in both business policy profiles, we obtain
$$\Pi_i(o(\hat{s})) = \frac{1}{1-\delta}\,\kappa_i\,\pi(p^{\mathrm{m}}) - f = \Pi_i(o(\tilde{s}))\ .$$
- If $\delta \geq \delta_{\mathrm{crit}}^{\tilde{g}}$ as well as $i \notin C(\hat{g})$ and $i \in C(\tilde{g})$, then both business policy profiles induce the collusive outcome. Since retailer i does not adopt a CC in business policy profile $\hat{s}$, but in business policy profile $\tilde{s}$, we obtain
$$\Pi_i(o(\hat{s})) = \frac{1}{1-\delta}\,\kappa_i\,\pi(p^{\mathrm{m}}) > \frac{1}{1-\delta}\,\kappa_i\,\pi(p^{\mathrm{m}}) - f = \Pi_i(o(\tilde{s}))\ .$$
- If $\delta < \delta_{\mathrm{crit}}^{\tilde{g}}$ as well as $i \in C(\hat{g})$ and $i \in C(\tilde{g})$, then different outcomes are realized by the two business policy profiles. While the collusive outcome is reached by business policy profile $\hat{s}$, the competitive outcome is reached by business policy profile $\tilde{s}$. Since retailer i adopts a CC in both business policy profiles, we obtain
$$\Pi_i(o(\hat{s})) = \frac{1}{1-\delta}\,\kappa_i\,\pi(p^{\mathrm{m}}) - f > -f = \Pi_i(o(\tilde{s}))\ .$$
- If $\delta < \delta_{\mathrm{crit}}^{\tilde{g}}$ as well as $i \notin C(\hat{g})$ and $i \in C(\tilde{g})$, then different outcomes are realized by the two business policy profiles. While the collusive outcome is reached by business policy profile $\hat{s}$, the competitive outcome is reached by business policy profile $\tilde{s}$. Since retailer i does not adopt a CC in business policy profile $\hat{s}$, but in business policy profile $\tilde{s}$, we obtain
$$\Pi_i(o(\hat{s})) = \frac{1}{1-\delta}\,\kappa_i\,\pi(p^{\mathrm{m}}) > -f = \Pi_i(o(\tilde{s}))\ .$$
- If $i \in C(\hat{g})$ and $i \notin C(\tilde{g})$, then $\tilde{g} = (w_i, \hat{g}_{-i})$. It follows from Property (M2) that $\delta < \delta_{\mathrm{crit}}^{\tilde{g}}$. Hence, while business policy profile $\hat{s}$ induces the collusive outcome, business policy profile $\tilde{s}$ induces the competitive outcome. Note that retailer i adopts a CC in business policy profile $\hat{s}$, but does not in business policy profile $\tilde{s}$. Nevertheless, we obtain from Property (M3) that
$$\Pi_i(o(\hat{s})) = \frac{1}{1-\delta}\,\kappa_i\,\pi(p^{\mathrm{m}}) - f = \frac{1}{1-\delta}\,\kappa_i\,\pi(p^{\mathrm{m}}) - f_{\mathrm{crit}}^{g,\delta} \geq 0 = \Pi_i(o(\tilde{s}))\ .$$

Summing up, we obtain $\Pi_i(o(\hat{s})) \geq \Pi_i(o(\tilde{s}))$ for any business policy profile $\tilde{s} := (\tilde{s}_i, \hat{s}_{-i})$ where $\tilde{s}_i := (\tilde{g}_i, (\hat{s}_i^g)_{g\in G})$. That means, Property (T2) has been established. Moreover, since business policy profile $\hat{s}$ also satisfies Property (T1) by construction, we conclude from Remark 3 that $\hat{s}$ is subgame perfect. As was shown above, $\hat{s}$ induces the collusive outcome. Hence, $\hat{s} \in \mathcal{S}^{\mathrm{m}}(\delta, f, n, z)$.

("*only if*") Suppose clause profile $\hat{g}$ is perfectly collusive. By definition, there is a business policy profile $\hat{s} := (\hat{g}, (\hat{s}^g)_{g\in G}) \in \mathcal{S}^{\mathrm{m}}(\delta, f, n, z)$. Obviously, $\hat{s} \in \mathcal{S}^{\mathrm{g}}(\delta, f, n, z)$. We conclude from Remark 3 that $\hat{s}$ satisfies Properties (T1) and (T2). Since $\hat{s}$ induces the collusive price outcome, it holds $\delta \geq \delta_{\mathrm{crit}}^{\hat{g}}$ and, hence, Property (M1) is satisfied.

We proceed indirectly to establish PROPERTY (M2). Let us suppose contrary to our claim that $\delta_{\text{crit}}^{\tilde{g}} \leq \delta$ for some clause profile $\tilde{g} := (w_i, \hat{g}_{-i})$ where $i \in C(\hat{g})$. We define business policy profile $\tilde{s} := (\tilde{g}, (\hat{s}^g)_{g \in G})$. Since business policy profile $\hat{s}^{\tilde{g}}$ induces the grim-trigger price policy profile in subgame $\Gamma^{\tilde{g}}$, the collusive outcome is realized by $\tilde{s}$ and it holds

$$\Pi_i(o(\tilde{s})) = \frac{1}{1-\delta}\,\kappa_i\,\pi(p^{\text{m}}) > \frac{1}{1-\delta}\,\kappa_i\,\pi(p^{\text{m}}) - f = \Pi_i(o(\hat{s}))\ .$$

However, this result contradicts PROPERTY (T2). Hence, it holds $\delta < \delta_{\text{crit}}^{\tilde{g}}$ for any $\tilde{g} := (w_i, \hat{g}_{-i})$ where $i \in C(\hat{g})$. We conclude that PROPERTY (M2) is satisfied.

We also proceed indirectly to establish PROPERTY (M3). Let us suppose that PROPERTIES (M1) and (M2) are satisfied and contrary to our claim that $f > f_{\text{crit}}^{\hat{g},\delta}$. We note that the latter strict inequality implies $C(\hat{g}) \neq \emptyset$; otherwise, it would be $f_{\text{crit}}^{\hat{g},\delta} = +\infty$ and the strict inequality could not be satisfied. Consider retailer $i := \min C(\hat{g})$ and business policy profile $\tilde{s} := (\tilde{s}_i, \hat{s}_{-i})$ where $\tilde{s}_i := (w_i, (\hat{s}_i^g)_{g \in G})$. Resorting to PROPERTIES (M1) and (M2), we obtain

$$\Pi_i(o(\tilde{s})) = 0 = \frac{1}{1-\delta}\,\kappa_i\,\pi(p^{\text{m}}) - f_{\text{crit}}^{\hat{g},\delta} > \frac{1}{1-\delta}\,\kappa_i\,\pi(p^{\text{m}}) - f = \Pi_i(o(\hat{s}))\ .$$

Apparently, this strict inequality violates PROPERTY (T2). Hence, it holds $f \leq f_{\text{crit}}^{\hat{g},\delta}$ and, thus, PROPERTY (M3) has been established. □

Proof of Proposition 5. We prove this claim by contradiction. Suppose $\hat{s} := (\hat{g}, (\hat{s}^g)_{g \in G})$ is a subgame perfect business policy profile (of $\mathcal{S}^{\text{g}}(\delta, f, n, z)$) in which retailer i adopts a CC, but which induces the competitive price outcome. That means, $\hat{g}_i \neq w_i$ and $\hat{s}^{\hat{g}} = c^{\hat{g}}$. Obviously, retailer i earns a profit $\Pi_i(o(\hat{s})) = -f$ at this business policy profile. Consider now the business policy profile $\tilde{s}$ which corresponds to $\hat{s}$ except that instead of adopting $\hat{g}_i$ retailer i abstains from offering any CC, i.e., $\tilde{s} := (\tilde{s}_i, \hat{s}_{-i})$ where $\tilde{s}_i := (w_i, (\hat{s}_i^g)_{g \in G})$. We obtain $\Pi_i(o(\tilde{s})) \geq 0 > \Pi_i(o(\hat{s}))$, violating property (T2). REMARK 3 implies that $\hat{s}$ is not subgame perfect. However, this result is at odds with our initial assumption. □

Proof of Theorem 6.

(a) We proceed indirectly. If $\delta < \delta_{\text{crit}}$, then clause profile w (i.e., the clause profile in which none of the retailers adopts a CC) violates PROPERTY (M1). We conclude from REMARK 4 that clause profile w is not perfectly collusive if $\delta < \delta_{\text{crit}}$.

(b) Suppose $\delta \geq \delta_{\text{crit}}$. Let us first examine whether clause profile w is perfectly collusive. By assumption, PROPERTY (M1) holds. PROPERTY (M2) is trivially satisfied because $C(w) = \emptyset$. Note that $f_{\text{crit}}^{w,\delta} = +\infty$ by definition. Therefore, PROPERTY (M3) is also satisfied. We infer from REMARK 4 that clause profile w is perfectly collusive.

To prove the uniqueness of the perfectly collusive clause profile, let us consider some clause profile $g \neq w$. Since $C(g) \neq \emptyset$, we can pick some $i \in C(g)$ and define clause profile $\tilde{g} := (w_i, g_{-i})$. Applying REMARK 2, we obtain $\delta \geq \delta_{\text{crit}}^{\tilde{g}}$. This in turn entails that clause profile g violates PROPERTY (M1). It follows from REMARK 4 that g is not perfectly collusive. Summing up, we have shown that if $\delta \geq \delta_{\text{crit}}$, then the trivial clause profile w is the unique perfectly collusive clause profile. □

Proof of Remark 7. Obviously, the claim is true for the trivial clause profile w. For this reason, let us suppose from now on that clause profile g satisfies $C(g) \neq \emptyset$. Our assumption $K_{C(g)} \leq D(p^{\text{m}})$ implies $k_1 \leq D(p^{\text{m}})$. Applying REMARK 2, we obtain $\delta_{\text{crit}} = 1 - \frac{D(p^{\text{m}})}{K}$.

Pick some $i \in C(g)$. We examine the situation in which retailer i advertises price $q_i < p^{\text{m}}$ while all of its competitors advertise price p^{m}. Since $K_{C(g)} \leq D(p^{\text{m}})$, retailer i's sales at the effective purchase prices q^{p} are $X_i(q^{\mathrm{p}}) = k_i$ by ASSUMPTIONS (D2), (R1), and (I). Hence, its profit amounts to $\pi_i^g(q_i, p_{-i}^{\mathrm{m}}) = (q_i - c)k_i$. Since $q_i < p^{\mathrm{m}}$ has been arbitrarily chosen, we have $\sup_{q_i < p^{\mathrm{m}}} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) = (p^{\mathrm{m}} - c)k_i$. It follows from REMARK 1(b) that $\sup_{q_i \neq p^{\mathrm{m}}} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) = (p^{\mathrm{m}} - c)k_i$ and, thus, $\delta_{i,\mathrm{crit}}^g = 1 - \frac{D(p^{\mathrm{m}})}{K}$. By REMARK 2, we obtain $\delta_{\mathrm{crit}}^g = \delta_{\mathrm{crit}}$. □

Proof of Theorem 8. We prove this claim by contradiction. Suppose there is a perfectly collusive clause profile $\hat{g}$ satisfying $K_{C(g)} \leq D(p^{\mathrm{m}})$. We already know from REMARK 7 that $\delta_{\mathrm{crit}}^g = \delta_{\mathrm{crit}}$. Since $\delta < \delta_{\mathrm{crit}}^g$ is assumed, PROPERTY (M1) is violated. It follows from REMARK 4 that clause profile $\hat{g}$ is not perfectly collusive. □

Proof of Remark 9. Pick some retailer i and suppose $\delta < \delta_{i,\mathrm{crit}}$. Due to ASSUMPTION (D2), the monopolistic profit mapping $\bar{\pi}_i(\cdot)$ is continuous. ASSUMPTIONS (D1) and (D3) ensure that $\bar{\pi}_i(\cdot)$ is increasing on $[c, p^{\mathrm{m}}]$. Moreover, it holds $\bar{\pi}_i(c) = 0$ and $\bar{\pi}_i(p^{\mathrm{m}}) = (p^{\mathrm{m}} - c)\min\{k_i, D(p^{\mathrm{m}})\}$. By assumption and REMARK 2, we have $\delta < 1 - \max\{\kappa_i, \frac{D(p^{\mathrm{m}})}{K}\}$. Regardless of whether $k_i \leq D(p^{\mathrm{m}})$ or $k_i > D(p^{\mathrm{m}})$, we always obtain

$$\bar{\pi}_i(c) = 0 < \frac{1}{1-\delta}\kappa_i \pi^{\mathrm{m}} < (p^{\mathrm{m}} - c)\min\{k_i, D(p^{\mathrm{m}})\} = \bar{\pi}_i(p^{\mathrm{m}}).$$

The INTERMEDIATE VALUE THEOREM implies that there exists some $\bar{p}_i^\delta \in]c, p^{\mathrm{m}}[$ satisfying $\bar{\pi}_i(\bar{p}_i^\delta) = \frac{1}{1-\delta}\kappa_i \pi^{\mathrm{m}}$. Moreover, since monopolistic profit mapping $\bar{\pi}_i(\cdot)$ is increasing on $[c, p^{\mathrm{m}}]$, value $\bar{p}_i^\delta$ is unique.

Consider some common discount factor $\beta < \delta$. As argued above, there exists some unique $\bar{p}_i^\beta \in]c, p^{\mathrm{m}}[$ satisfying $\bar{\pi}_i(\bar{p}_i^\beta) = \frac{1}{1-\beta}\kappa_i \pi^{\mathrm{m}}$. Note that $\frac{1}{1-\beta}\kappa_i \pi^{\mathrm{m}} < \frac{1}{1-\delta}\kappa_i \pi^{\mathrm{m}}$. Since monopolistic profit mapping $\bar{\pi}_i(\cdot)$ is increasing on $[c, p^{\mathrm{m}}]$, we obtain $\bar{p}_i^\beta < \bar{p}_i^\delta$.

Consider some retailer $j < i$. As argued above, there exists some unique $\bar{p}_i^\delta \in]c, p^{\mathrm{m}}[$ so that $\bar{\pi}_i(\bar{p}_i^\delta) = \frac{1}{1-\delta}\kappa_i \pi^{\mathrm{m}}$. Since $k_j < k_i$, it holds $\bar{\pi}_j(p) \geq \frac{k_j}{k_i}\bar{\pi}_i(p)$ for any $p \geq c$. In particular, we have $\bar{\pi}_j(\bar{p}_i^\delta) \geq \frac{k_j}{k_i}\bar{\pi}_i(\bar{p}_i^\delta) = \frac{1}{1-\delta}\kappa_j \pi^{\mathrm{m}}$. Since monopolistic profit mapping $\bar{\pi}_j(\cdot)$ is increasing on $[c, p^{\mathrm{m}}]$, we conclude that $\bar{p}_j^\delta \leq \bar{p}_i^\delta$. □

Proof of Theorem 10. We prove this claim by contradiction. Suppose that $f > \bar{f}^\delta$, but $\hat{g}$ is perfectly collusive. Since $\delta < \delta_{\mathrm{crit}}$, we infer from THEOREM 8 that $K_{C(\hat{g})} > D(p^{\mathrm{m}})$. Obviously, $C(\hat{g}) \neq \emptyset$ and, thus, we can define $i := \min C(\hat{g})$. By assumption, $f > \frac{1}{1-\delta}\kappa_i \pi_i(p^{\mathrm{m}})$. However, this strict inequality is at odds with PROPERTY (M3). It follows from REMARK 4 that $\hat{g}$ is not perfectly collusive.

From now on, we assume that $f \leq \bar{f}^\delta$, but $z > \bar{z}_1^\delta$. To show that there is no perfectly collusive collusive price even in this case, consider some arbitrary clause profile g and retailer 1. Moreover, suppose that retailer 1 advertises some price $c + \bar{z}_1^\delta < q_1 < \min\{c + z, p^{\mathrm{m}}\}$ while its competitors advertise price p^{m}. Due to ASSUMPTION (G), it holds $q_1 < q_j^p$ for any retailer j different from 1. Therefore, we obtain

$$\pi_1^g(q_1, p_{-1}^{\mathrm{m}}) = (q_1 - c)\min\{k_1, D(q_1)\} = \bar{\pi}_1(q_1)$$

By definition, $\bar{\pi}_1(c + \bar{z}_1^\delta) = \frac{1}{1-\delta}\kappa_1 \pi^{\mathrm{m}}$. Since $\bar{\pi}_1(\cdot)$ is increasing on $[c, p^{\mathrm{m}}]$, it follows $\pi_1^g(q_1, p_{-1}^{\mathrm{m}}) > \frac{1}{1-\delta}\kappa_1 \pi^{\mathrm{m}}$ and, thus,

$$\sup_{q_1 \neq p^{\mathrm{m}}} \pi_1^g(q_1, p_{-1}^{\mathrm{m}}) > \frac{1}{1-\delta}\kappa_1 \pi^{\mathrm{m}}.$$

We conclude from this result that $\delta < \delta_{1,\mathrm{crit}}^g \leq \delta_{\mathrm{crit}}^g$. However, this results violates PROPERTY (M1) of REMARK 4. Consequently, clause profile g is not perfectly collusive. □

Proof of Theorem 11. Consider a clause profile $\hat{g}$ in which retailer i offers a CC of the form $\hat{g}_i^{e,\mu}$ where $\mu < z$. Moreover, we define $g := (w_i, \hat{g}_{-i})$, which is the clause profile corresponding to $\hat{g}$ except from retailer i not implementing a CC. We will show $\delta_{\mathrm{crit}}^{\hat{g}} = \delta_{\mathrm{crit}}^g$.

For this purpose, let us first consider retailer i. We suppose that this retailer advertises price $q_i \neq p^{\mathrm{m}}$ while its competitors advertise price p^{m}. In the following, we compare the profits retailer i earns under the clause profiles $\hat{g}$ and g given such price announcements.

- Suppose $q_i > p^{\mathrm{m}}$. By assumption, it holds $p^{\mathrm{m}} - z < \hat{g}_i^{\mathrm{s}}(q_i, p_{-i}^{\mathrm{m}}) \leq g_i^{\mathrm{s}}(q_i, p_{-i}^{\mathrm{m}}) = q_i$. This immediately implies $\hat{g}_i^{\mathrm{p}}(q_i, p_{-i}^{\mathrm{m}}) > p^{\mathrm{m}}$ and $g_i^{\mathrm{p}}(q_i, p_{-i}^{\mathrm{m}}) > p^{\mathrm{m}}$. Moreover, due to Assumption (G), we have $\hat{g}_k^{\mathrm{p}}(q_i, p_{-i}^{\mathrm{m}}) = g_k^{\mathrm{p}}(q_i, p_{-i}^{\mathrm{m}}) = p^{\mathrm{m}}$ for any $k \neq i$. We conclude from Assumptions (D2), (D4), (R1), and (I) that $X_i(\hat{g}^{\mathrm{p}}(q_i, p_{-i}^{\mathrm{m}})) = 0 = X_i(g^{\mathrm{p}}(q_i, p_{-i}^{\mathrm{m}}))$ and, thus, $\pi_i^{\hat{g}}(q_i, p_{-i}^{\mathrm{m}}) = 0 = \pi_i^{g}(q_i, p_{-i}^{\mathrm{m}})$.
- Suppose $q_i < p^{\mathrm{m}}$. By Assumption (G), we have $\hat{g}_i^{\mathrm{p}}(q_i, p_{-i}^{\mathrm{m}}) = g_i^{\mathrm{p}}(q_i, p_{-i}^{\mathrm{m}}) = q_i$. Moreover, it holds $\hat{g}_k^{\mathrm{p}}(q_i, p_{-i}^{\mathrm{m}}) = g_k^{\mathrm{p}}(q_i, p_{-i}^{\mathrm{m}}) \leq p^{\mathrm{m}}$ for any $k \neq i$. Since $\hat{g}^{\mathrm{p}}(q_i, p_{-i}^{\mathrm{m}}) = g^{\mathrm{p}}(q_i, p_{-i}^{\mathrm{m}})$, we obtain $X_i(\hat{g}^{\mathrm{p}}(q_i, p_{-i}^{\mathrm{m}}) = X_i(g^{\mathrm{p}}(q_i, p_{-i}^{\mathrm{m}}))$. It follows from $\hat{g}_i^{\mathrm{s}}(q_i, p_{-i}^{\mathrm{m}}) = g_i^{\mathrm{s}}(q_i, p_{-i}^{\mathrm{m}}) = q_i$ that $\pi_i^{\hat{g}}(q_i, p_{-i}^{\mathrm{m}}) = \pi_i^{g}(q_i, p_{-i}^{\mathrm{m}})$.

Let us now consider some retailer $j \neq i$ and suppose that this retailer advertises price $q_j \neq p^{\mathrm{m}}$ while its competitors advertise price p^{m}. In the following, we will specify the profits retailer j earns under the clause profiles $\hat{g}$ and g given such price announcements.

- Suppose $q_j > p^{\mathrm{m}}$. It holds $\hat{g}_j^{\mathrm{p}}(q_j, p_{-j}^{\mathrm{m}}) = g_j^{\mathrm{p}}(q_j, p_{-j}^{\mathrm{m}}) \leq q_k$. Moreover, Assumption (G) implies $\hat{g}_j^{\mathrm{p}}(q_j, p_{-j}^{\mathrm{m}}) = g_j^{\mathrm{c}}(q_j, p_{-j}^{\mathrm{m}}) = p^{\mathrm{m}}$ for any $k \neq j$. Since $\hat{g}^{\mathrm{p}}(q_j, p_{-j}^{\mathrm{m}}) = g^{\mathrm{p}}(q_j, p_{-j}^{\mathrm{m}})$, we obtain $X_j(\hat{g}^{\mathrm{p}}(q_j, p_{-j}^{\mathrm{m}}) = X_k(g^{\mathrm{p}}(q_j, p_{-j}^{\mathrm{m}}))$. It follows from $\hat{g}_i^{\mathrm{s}}(q_i, p_{-i}^{\mathrm{m}}) = g_j^{\mathrm{s}}(q_j, p_{-j}^{\mathrm{m}})$ that $\pi_j^{\hat{g}}(q_j, p_{-j}^{\mathrm{m}}) = \pi_j^{g}(q_j, p_{-j}^{\mathrm{m}})$.
- Suppose $q_j < p^{\mathrm{m}}$. We note that $q_j = \hat{g}_j^{\mathrm{p}}(q_j, p_{-j}^{\mathrm{m}}) = g_j^{\mathrm{p}}(q_j, p_{-j}^{\mathrm{m}})$. Moreover, it holds $\hat{g}_k^{\mathrm{p}}(q_k, p_{-k}^{\mathrm{m}}) = g_k^{\mathrm{p}}(q_k, p_{-k}^{\mathrm{m}}) \leq p^{\mathrm{m}}$ for any $k \neq i, j$. By assumption, we have $q_j - z < \hat{g}_i^{\mathrm{s}}(q_i, p_{-i}^{\mathrm{m}}) \leq g_i^{\mathrm{s}}(q_i, p_{-i}^{\mathrm{m}}) = p^{\mathrm{m}}$. This in turn implies $q_j < \hat{g}_i^{\mathrm{p}}(q_i, p_{-i}^{\mathrm{m}}) \leq g_i^{\mathrm{p}}(q_i, p_{-i}^{\mathrm{m}}) = p^{\mathrm{m}}$. Hence, we obtain $[\hat{g}^{\mathrm{p}}(q_j, p_{-j}^{\mathrm{m}}) < q_j] = [g_j^{\mathrm{p}}(q_j, p_{-j}^{\mathrm{m}}) < q_j]$ and $[\hat{g}^{\mathrm{p}}(q_j, p_{-j}^{\mathrm{m}}) = q_j] = [g_j^{\mathrm{p}}(q_j, p_{-j}^{\mathrm{m}}) = q_j]$. We conclude from Assumptions (R2) and (I) that $X_j(\hat{g}^{\mathrm{p}}(q_j, p_{-j}^{\mathrm{m}}) = X_k(g^{\mathrm{p}}(q_j, p_{-j}^{\mathrm{m}}))$. It follows from $\hat{g}_i^{\mathrm{s}}(q_i, p_{-i}^{\mathrm{m}}) = g_j^{\mathrm{s}}(q_j, p_{-j}^{\mathrm{m}})$ that $\pi_j^{\hat{g}}(q_j, p_{-j}^{\mathrm{m}}) = \pi_j^{g}(q_j, p_{-j}^{\mathrm{m}})$.

Summing up, we have shown $\pi_k^{\hat{g}}(q_k, p_{-k}^{\mathrm{m}}) = \pi_k^{g}(q_k, p_{-k}^{\mathrm{m}})$ for any $q_k \neq p^{\mathrm{m}}$ and any $k \in I$. This implies $\sup_{q_k \neq p^{\mathrm{m}}} \pi_k^{\hat{g}}(q_k, p_{-k}^{\mathrm{m}}) = \sup_{q_k \neq p^{\mathrm{m}}} \pi_k^{g}(q_k, p_{-k}^{\mathrm{m}})$ and, thus, $\delta_{j,\mathrm{crit}}^{\hat{g}} = \delta_{j,\mathrm{crit}}^{g}$ for any $j \in I$. The latter result in turn entails $\delta_{\mathrm{crit}}^{\hat{g}} = \delta_{\mathrm{crit}}^{g}$.

However, the equality violates Property (M2). Since according to Remark 4 this property is a necessary condition for subgame perfect collusion, we conclude that clause profile $\hat{g}$ is not perfectly collusive. Thereby, we have shown that none of the retailers in a perfectly collusive profile adopts a CC with lump sum refund $\mu < z$. ☐

Proof of Remark 12.

Before proving parts (a) and (b) of the claim, we derive some general statements. First, note that if $\kappa_J \geq 1 - \delta$ and $\delta < \delta_{\mathrm{crit}}$, it follows from Remark 2 that $\kappa_J > \max\left\{\kappa_1, \frac{D(p^{\mathrm{m}})}{K}\right\}$. This in turn implies $K_J > D(p^{\mathrm{m}})$. Therefore, although not explicitly stated there, $K_J > D(p^{\mathrm{m}})$ holds even in part (b).

Let g be a clause profile in which a coalition $J \subseteq I$ of retailers adopt a CC with lump sum refund z and the other retailers do not adopt a CC, i.e., g is given by $g := (g_J, w_{-J})$ where $g_i := g_i^{€,z}$ for any $i \in J$. We suppose that $K_J > D(p^{\mathrm{m}})$.

Pick some $i \in J$ of the CC-adopting retailers and consider the situation in which retailer i advertises price $q_i \neq p^{\mathrm{m}}$ while its competitors advertise price p^{m}. In the following, we will provide some upper bounds of i's profit $\pi_i^{g}(q_i, p_{-i}^{\mathrm{m}})$.

- If $q_i > p^{\mathrm{m}}$, the CC of retailer i implies $q_i^{\mathrm{p}} = p^{\mathrm{m}}$ and $q_i^{\mathrm{s}} = p^{\mathrm{m}} - z$. Moreover, we have $q_j^{\mathrm{p}} = q_i^{\mathrm{s}} = p^{\mathrm{m}}$ for any competitor $j \neq i$. It follows from Assumptions (R1) and (I) that retailer i's sales are $X_i(q^{\mathrm{p}}) = \kappa_i D(p^{\mathrm{m}})$ and, thus, its profit amounts to $\pi_i^g(q_i, p_{-i}^{\mathrm{m}}) = (p^{\mathrm{m}} - z - c)\, \kappa_i\, D(p^{\mathrm{m}})$. Since the latter result holds for any $q_i > p^{\mathrm{m}}$, we obtain
$$\sup_{q_i > p^{\mathrm{m}}} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) = (p^{\mathrm{m}} - z - c)\, \kappa_i\, D(p^{\mathrm{m}})\ .$$
- If $c + z \leq q_i < p^{\mathrm{m}}$, it holds $q_i^{\mathrm{p}} = q_i^{\mathrm{s}} = q_i$. Moreover, the CCs of the competitors $j \in J \setminus \{i\}$ imply that $q_j^{\mathrm{s}} = q_i - z$ and $q_j^{\mathrm{p}} = q_i$ for any $j \in J \setminus \{i\}$. Obviously, the competitors $j \in I \setminus J$ charge price $q_j^{\mathrm{p}} = q_j^{\mathrm{s}} = p^{\mathrm{m}}$. It follows from Assumptions (R1) and (I) that retailer i's sales are $X_i(q^{\mathrm{p}}) = \min\{k_i, \frac{\kappa_i}{\kappa_J} D(q_i)\}$ so that it earns a profit $\pi_i^g(q_i, p_{-i}^{\mathrm{m}}) = (q_i - c) \min\{k_i, \frac{\kappa_i}{\kappa_J} D(q_i)\}$. Since the latter result holds for any $c + z \leq q_i < p^{\mathrm{m}}$ and $K_J > D(p^{\mathrm{m}})$ is assumed, we conclude from Assumptions (D1), (D2), and (D3) that
$$\sup_{c+z \leq q_i < p^{\mathrm{m}}} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) = (p^{\mathrm{m}} - c)\, \frac{\kappa_i}{\kappa_J}\, D(p^{\mathrm{m}})\ .$$
- If $c \leq q_i < c + z$, it holds $q_i^{\mathrm{p}} = q_i^{\mathrm{s}} = q_i$. Moreover, since the CCs of the competitors $j \in J \setminus \{i\}$ satisfy Assumption (G), we have $q_j^{\mathrm{s}} = c$ and $q_j^{\mathrm{p}} = c + z > q_i$ for any competitor $j \in J \setminus \{i\}$. Obviously, the competitors $j \in I \setminus J$ charge price $q_j^{\mathrm{p}} = q_j^{\mathrm{s}} = p^{\mathrm{m}}$. It follows from Assumptions (R1) and (I) that retailer i's sales are $X_i(q^{\mathrm{p}}) = \min\{k_i, D(q_i)\}$ and, thus, its profit amounts to $\pi_i^g(q_i, p_{-i}^{\mathrm{m}}) = \bar{\pi}_i(q_i) = (q_i - c) \min\{k_i, D(q_i)\}$. Since the latter result holds for any $c \leq q_i < c + z$ and monopolistic profit mapping $\bar{\pi}_i(\cdot)$ is increasing on $[c, p^{\mathrm{m}}]$ and continuous due to Assumptions (D1), (D2), and (D3), we obtain
$$\sup_{c \leq q_i < c+z} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) = z\, \min\{k_i, D(c + z)\}\ .$$

Pick now some retailer $i \notin J$ and suppose that it advertises price $q_i \neq p^{\mathrm{m}}$ while its competitors advertise price p^{m}. In the following, we will provide some upper bounds of i's profit $\pi_i^g(q_i, p_{-i}^{\mathrm{m}})$.

- If $q_i > p^{\mathrm{m}}$, then $q_i^{\mathrm{p}} = q_i^{\mathrm{s}} = q_i$. Moreover, we have $q_j^{\mathrm{p}} = q_i^{\mathrm{s}} = p^{\mathrm{m}}$ for any competitor $j \neq i$. It follows from Assumptions (D4), (R1), and (I) that retailer i has no sales and, thus, its profit is $\pi_i^g(q_i, p_{-i}^{\mathrm{m}}) = 0$. Since the latter result holds for any $q_i < p^{\mathrm{m}}$, we conclude
$$\sup_{q_i > p^{\mathrm{m}}} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) = 0.$$
- If $c + z \leq q_i < p^{\mathrm{m}}$, it holds $q_i^{\mathrm{p}} = q_i^{\mathrm{s}} = q_i$. Moreover, the CCs of the competitors $j \in J$ imply that $q_j^{\mathrm{s}} = q_i - z$ and $q_j^{\mathrm{p}} = q_i$ for any $j \in J$. The other competitors $j \in I \setminus (J \cup \{i\})$ charge price $q_j^{\mathrm{p}} = q_j^{\mathrm{s}} = p^{\mathrm{m}}$. It follows from Assumptions (R1) and (I) that retailer i's sales are $X_i(q^{\mathrm{p}}) = \min\{k_i, \frac{\kappa_i}{\kappa_{J\cup\{i\}}} D(q_i)\}$ so that it earns a profit $\pi_i^g(q_i, p_{-i}^{\mathrm{m}}) = (q_i - c) \min\{k_i, \frac{\kappa_i}{\kappa_{J\cup\{i\}}} D(q_i)\}$. Since the latter result holds for any $c + z \leq q_i < p^{\mathrm{m}}$ and $K_J > D(p^{\mathrm{m}})$ is assumed, we conclude from Assumptions (D1), (D2), and (D3) that
$$\sup_{c+z \leq q_i < p^{\mathrm{m}}} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) = (p^{\mathrm{m}} - c)\, \frac{\kappa_i}{\kappa_{J\cup\{i\}}}\, D(p^{\mathrm{m}})\ .$$
- If $c \leq q_i < c + z$, it holds $q_i^{\mathrm{p}} = q_i^{\mathrm{s}} = q_i$. Since the CCs of the competitors $j \in J$ satisfy Assumption (G), we have $q_j^{\mathrm{s}} = c$ and $q_j^{\mathrm{p}} = c + z > q_i$ for any $j \in J$. The other competitors $j \in I \setminus J$ charge price $q_j^{\mathrm{p}} = q_j^{\mathrm{s}} = p^{\mathrm{m}}$. It follows from Assumptions (R1) and (I) that retailer i's sales are $X_i(q^{\mathrm{p}}) = \min\{k_i, D(q_i)\}$ and, thus, its profit amounts to $\pi_i^g(q_i, p_{-i}^{\mathrm{m}}) = \bar{\pi}_i(q_i) = (q_i - c) \min\{k_i, D(q_i)\}$. Since the latter result holds for any $c \leq q_i < c + z$ and monopolistic profit mapping $\bar{\pi}_i(\cdot)$ is increasing on $[c, p^{\mathrm{m}}]$ and continuous due to Assumptions (D1), (D2), and (D3), we obtain
$$\sup_{c \leq q_i < c+z} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) = z\, \min\{k_i, D(c + z)\}\ .$$

(a) Consider some $i \in J$. We infer from our previous calculations that

$$\sup_{q_i \neq p^{\mathrm{m}}} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) \geq \frac{\kappa_i}{\kappa_J}\, \pi^{\mathrm{m}} \ .$$

This in turn entails $\delta_{i,\mathrm{crit}}^g \geq 1 - \kappa_J$. Therefore, it holds $\delta_{\mathrm{crit}}^g \geq 1 - \kappa_J$.

Consider now some arbitrary $i \in I$, but suppose $z \leq \bar{z}_1^{1-\kappa_J}$. Note that $\bar{\pi}_i(p) \leq \frac{\kappa_i}{\kappa_1}\bar{\pi}_1(p)$ for any $p \in \mathbb{R}_+$. Since $\bar{\pi}_1(c + \bar{z}_1^{1-\kappa_J}) = \frac{\kappa_1}{\kappa_J}\pi^{\mathrm{m}}$ by definition, it holds $\bar{\pi}_i(c + \bar{z}_1^{1-\kappa_J}) \leq \frac{\kappa_i}{\kappa_J}\pi^{\mathrm{m}}$. Due to Assumptions (D2) and (D3), monopolistic profit mapping $\bar{\pi}_1(\cdot)$ is increasing on $[c, p^{\mathrm{m}}]$. Therefore, we obtain

$$\sup_{c \leq q_i < c+z} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) \leq \frac{\kappa_i}{\kappa_J}\, \pi^{\mathrm{m}}.$$

This, together with our previous calculations and Remark 1(a), entails $\sup_{q_i \neq p^{\mathrm{m}}} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) \leq \frac{\kappa_i}{\kappa_J}\, \pi^{\mathrm{m}}$ and, thus, $\delta_{i,\mathrm{crit}}^g \leq 1-\kappa_J$ for any $i \in I$. We already know that $\delta_{i,\mathrm{crit}}^g \geq 1-\kappa_J$ for any $i \in J$. Therefore, $\delta_{i,\mathrm{crit}}^g = 1 - \kappa_J$ for any $i \in J$. We conclude from this result that $\delta_{\mathrm{crit}}^g = 1 - \kappa_J$ if $z \leq \bar{z}_1^{1-\kappa_J}$.

(b) Consider some $i \in I$ and suppose $z \leq \bar{z}_1^{\delta}$. Since $\kappa_J \geq 1 - \delta$ is also assumed, it follows from our previous calculations that

$$\sup_{c+z \leq q_i < p^{\mathrm{m}}} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) \leq \frac{\kappa_i}{\kappa_J}\pi^{\mathrm{m}} \leq \frac{\kappa_i}{1-\delta}\pi^{\mathrm{m}}$$

where both weak inequalities turn into equalities if, and only if, $\kappa_J = 1 - \delta$ and $i \in J$.

We remark that $\bar{\pi}_i(p) \leq \frac{\kappa_i}{\kappa_1}\bar{\pi}_1(p)$ for any $p \in \mathbb{R}_+$. Since $\bar{\pi}_1(c+\bar{z}_1^{\delta}) = \frac{\kappa_1}{1-\delta}\pi^{\mathrm{m}}$ by definition, it holds $\bar{\pi}_i(c + \bar{z}_1^{\delta}) \leq \frac{\kappa_i}{1-\delta}\pi^{\mathrm{m}}$. Since monopolistic profit mapping $\bar{\pi}_i(\cdot)$ is increasing on $[c, p^{\mathrm{m}}]$ and continuous due to Assumptions (D1), (D2) and (D3), it follows

$$\sup_{c \leq q_i < c+z} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) \leq \frac{\kappa_i}{1-\delta}\, \pi^{\mathrm{m}}$$

where the weak inequality becomes strict if $z < \bar{z}_1^{\delta}$ and turns into the equality if $z = \bar{z}_1^{\delta}$ and $i = 1$.

Summing up, the above findings and Remark 1(a) imply

$$\sup_{q_i \neq p^{\mathrm{m}}} \pi_i^g(q_i, p_{-i}^{\mathrm{m}}) \leq \frac{\kappa_i}{1-\delta}\, \pi^{\mathrm{m}}$$

for any $i \in I$. This in turn entails $\delta_{i,\mathrm{crit}}^g \leq \delta$ for any $i \in I$ and, thus, $\delta_{\mathrm{crit}}^g \leq \delta$. Moreover, we conclude from the above findings that $\delta_{\mathrm{crit}}^g = \delta$ if, and only if, $\kappa_J = 1 - \delta$ or $z = \bar{z}_i^{\delta}$. $\square$

Proof of Theorem 13. Suppose our competition model $\Gamma(\delta, f, n, z)$ satisfies conditions $\delta < \delta_{\mathrm{crit}}$ and $z \leq \bar{z}_1^{\delta}$. The latter condition means that the hassle-free case $z = 0$ as well as the case with hassle costs $0 < z < \bar{z}_1^{\delta}$ are taken into account.

(*"if"*) Consider a non-trivial clause profile $\hat{g} := (\hat{g}_J^{€,z}, w_{-J})$ satisfying conditions (i) , (ii), and (iii). We apply the characterization in Remark 4 to establish that $\hat{g}$ is perfectly collusive. Note that condition (iii) is nothing but Property (M3). Therefore, it remains to prove Properties (M1) and (M2).

Applying condition (i) and Remark 12(b), we obtain $\delta_{\mathrm{crit}}^{\hat{g}} \leq \delta$ and, thus, Property (M1). To prove Property (M2), pick some $i \in J$ and consider clause profile $g := (w_i, \hat{g}_{-i})$. If $K_{J\setminus\{i\}} \leq D(p^{\mathrm{m}})$, we resort to Remark 7 and immediately obtain $\delta_{\mathrm{crit}}^g = \delta_{\mathrm{crit}} > \delta$. If $K_{J\setminus\{i\}} > D(p^{\mathrm{m}})$, we apply Remark 12(a). This remark implies $\delta_{\mathrm{crit}}^g \geq 1-\kappa_{J\setminus\{i\}}$. Define $j := \min J$. Since $1-\kappa_{J\setminus\{i\}} \geq 1-\kappa_{J\setminus\{j\}}$ and $1-\kappa_{J\setminus\{j\}} > \delta$ due to condition (ii), we finally obtain $\delta_{\mathrm{crit}}^g > \delta$. Summing up, we have established Properties (M1), (M2), and (M3). Hence, $\hat{g}$ proves to be perfectly collusive.

(*"only if"*) Consider a non-trivial and perfectly collusive clause profile $\hat{g} := (\hat{g}_J^{€,z}, w_{-J})$. According to Remark 4, clause profile $\hat{g}$ satisfies Properties (M1), (M2), and (M3). Note that the latter property is identical to condition (iii). Therefore, it remains to prove conditions (i) and (ii).

Since $\hat{g}$ is perfectly collusive and $\delta < \delta_{\text{crit}}$, THEOREM 8 requires $K_J > D(p^{\text{m}})$. It then follows from REMARK 12(a) that $1 - \kappa_J \leq \delta^{\hat{g}}_{\text{crit}}$. Since $\delta^{\hat{g}}_{\text{crit}} \leq \delta$ by PROPERTY (M1), we obtain $1 - \kappa_J \leq \delta$ and, thus, condition (i). Define $j := \min J$ and consider clause profile $g := (w_j, \hat{g}_{-j})$. Since $\delta^{g}_{\text{crit}} > \delta$ by PROPERTY (M2), we deduce from REMARK 12(b) that $\kappa_{J\setminus\{j\}} < 1 - \delta$. That means, condition (ii) also holds. Summing up, we have shown that conditions (i), (ii), and (iii) are satisfied. $\square$

Proof of Proposition 15. Pick some common discount factor $\delta \leq \delta_{\text{crit}}$ and define

$$\mathscr{J} := \left\{ J \subseteq I : (g^{€,z}_J, w_{-J}) \text{ is perfectly collusive in } \Gamma(\delta, f, n, z) \right\} .$$

Obviously, $\hat{k} := \max\{k \in I : 1 - \kappa_{I_k} \leq \delta\}$ is well-defined. We stipulate $\hat{J} := I_{\hat{k}}$. Since clause profile $\hat{g} := (g^{€,z}_{\hat{J}}, w_{-\hat{J}})$ satisfies the conditions (i) - (iii) of THEOREM 13, it is perfectly collusive in $\Gamma(\delta, f, n, z)$. Therefore, $\mathscr{J} \neq \emptyset$. We will demonstrate that $\hat{g}$ is the only robustly collusive clause profile in $G^{€,z}$.

Consider first some clause profile $g := (g^{€,z}_J, w_{-J}) \in G^{€,z}$ where $|J| < |\hat{J}|$. Condition (ii) of THEOREM 13 and the construction of $\hat{k}$ imply $\delta < 1 - \kappa_{\hat{J}\setminus\{\hat{k}\}} \leq 1 - \kappa_J$. That means, g does not satisfy condition (i) of THEOREM 13. In consequence, g is not perfectly collusive. Since any clause profile $g := (g^{€,z}_J, w_{-J}) \in G^{€,z}$ satisfying $|J| < |\hat{J}|$ is not perfectly collusive, we have shown that $\hat{g}$ is cost-efficient in $G^{€,z}$.

Let us now consider a clause profile $g := (g^{€,z}_J, w_{-J}) \in G^{€,z}$ where $J \in \mathscr{J}$ and $|J| = |\hat{J}|$, but $J \neq \hat{J}$. That is, g is a perfectly collusive clause profile of $G^{€,z}$ which is different from $\hat{g}$, but has the same number of CC-adopting retailers. Obviously, we have $1 - \kappa_{\hat{J}} < 1 - \kappa_J$. It then follows from REMARK 9 that $\bar{z}^0_1 < \bar{z}^{1-\kappa_{\hat{J}}}_1 < \bar{z}^{1-\kappa_J}_1$. Applying REMARK 12(a), we obtain $\delta^{\hat{g}}_{\text{crit}} = 1 - \kappa_{\hat{J}} < 1 - \kappa_J = \delta^{g}_{\text{crit}}$. Let us stipulate $j := \min J$. By assumption, it holds $j < \hat{k}$. This in turn implies $f^{\delta,g}_{\text{crit}} = \frac{1}{1-\delta}\kappa_j \pi^{\text{m}} < \frac{1}{1-\delta}\kappa_{\hat{k}} \pi^{\text{m}} = f^{\delta,\hat{g}}_{\text{crit}}$. Summing up, we have shown that $\hat{g}$ has a lower critical discount factor and greater critical implementation costs than g. We conclude from these results that $\hat{g}$ is the only robustly collusive clause profile in $G^{€,z}$. $\square$

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University of Hohenheim
Dean’s Office of the Faculty of Business, Economics and Social Sciences
Palace Hohenheim 1 B
70593 Stuttgart | Germany
Fon +49 (0)711 459 22488
Fax +49 (0)711 459 22785
wiso@uni-hohenheim.de
wiso.uni-hohenheim.de